EDITORIAL

This whole issue of the Colloquium Journal, v. IX is devoted to the topics and the type of papers we usually publish in the Educational Resources section. In the past, the section contained papers that described the experiences you gained from formal course study and/or through your own teaching. In this issue this tradition is maintained.

Alex Ballantyne described the development of his understanding of the work of Vygotsky and Piaget that evolved throughout the Reasoning and Problem Solving in Mathematics and Science course. Alex was interested in the context of the emergence of language in the child and its relationship to thought. In the paper he articulated some differences in the perspectives of the two scholars and stressed the importance of the role that language skills play in the development of the ability of the child to reason. He synergistically examined the perspectives on individual cognitive development and the role of cultural and social transfer of information that leads to the construction of knowledge.

As the last class in their course program, Jessica Devonis and Kate McLaughlin undertook an independent study focusing on Qualitative Research. Naturalistic Inquiry by Yvonna S. Lincoln and Egon G. Guba was the primary reading material for this course. In their papers, Jessica and Kate shared about the course outcomes.

Jessica investigated the attitudes of college mathematics students toward homework. She aimed to understand students' motivation toward homework and their study habits. In carrying out the study, she held separate interviews with four college students who were taking Algebra and Trigonometry summer course at a state college. While learning how to conduct qualitative study, Jessica found that most students liked the on-line type of homework, and wanted homework to be collected and graded, but realized that they do not spend enough time on homework.

Kate wanted to develop a deeper understanding of naturalistic inquiry, the process of designing research questions and defining appropriate methods to study those questions. The specific focus of her pilot study was to gauge urban elementary teachers' perceptions of the implementation process of TERC’s (Teacher Education Research Center, Cambridge, MA) Investigations in Number, Data, and Space. Kate interviewed three teachers and analyzed their responses in light of the concept of pedagogical content knowledge. She concluded that district teacher professional development plans must include more instruction focused on mathematics, pedagogy, and teacher support on a regular basis throughout the school year.

Sharyn Gallagher summarized her experience as a finance instructor in developing and using graphic in a graduate finance course. She described the process of developing and teaching the course that incorporated graphic to illustrate key concepts taught and their interrelationships. Sharyn stressed that graphic provides a visual framework for the students to grasp the major focus of the course. She concluded that, if developed well, graphic could facilitate learning and retention of information and details.

Andy Golay is a young talented mathematics teacher. During the summer of 2004 he had an opportunity to work for the Education Development Center (EDC) in Newton, MA, and to participate in the development of a high-school curriculum. In his article Andy shared some of the main features of the curriculum that contains four courses: Algebra 1, Geometry, Algebra 2, and an equivalent to pre-calculus. The Algebra 1 course is being field-tested this year nationwide by teachers in several states, including Massachusetts, Colorado, California, and Washington State. In the article Andy modeled student dialogues and described the use of different representations of mathematical concepts.

Peggy LaBrosse examined whether the sequence of high school science courses in her school — physical science, biology, chemistry, and physics — matters in promoting student understanding of science concepts and achieving scientific literacy. Peggy indicated that in the present sequence of science courses, ninth graders take physical science, tenth graders take biology, eleventh graders take chemistry, and twelfth graders take physics. Peggy asserted that the sequence is inappropriate and does not respect the development in the disciplines over the past century. She made recommendations for the sequence of courses and provided alternative suggestions for increasing the number of science students.

Rocco Perla and Danielle M. Cross continuously search for better ways to refine and describe their inquiry into two philosophical schools, logical positivism and postpositivism.

This article extends and elaborates on their previous work published in volume VIII and presented at last year’s Colloquium. Rocco and Danielle responded to a number of valuable criticisms and comments made by their peers. During the presentation and subsequent discussions they realized that a number of concepts central to the viability of the mathematical models were imprecise and difficult to understand, and that the key definitions were rather cumbersome. In this article they provided the first mathematical proof of the popular scholarly conjecture that models of positivism and postpositivism are qualitatively different. They also argued that the use of precise language and structure of qualitative mathematics could benefit some learners in certain learning environments. Their work is a salient example of how the colloquium encourages critical analysis and sustained discourse among educational researchers.

If you want to exercise your analytical and writing skills, the journal is a great opportunity.

Regina M. Panasuk
GUIDELINES FOR SUBMISSION

The papers submitted for the Journal must discuss psychological and pedagogical issues and trends related to mathematics and science education.

WHEN SUBMITTING A PAPER, PLEASE USE THE FOLLOWING GUIDELINES:

1. Submit an electronic version of the paper and one hard copy, an abstract, approximately 150 words, and a biographical sketch, about 30 words. All pictures and diagrams must be submitted in a separate document.

2. Use double spacing with one-inch margins.

3. For references, tables and figures follow the style described in the Publication Manual of the American Psychological Association (APA), Fifth Edition.

4. Paper length should not exceed 30 pages, including pictures, tables, figures, and list of references.

5. Paper must be received by November 15.

6. Authors will be notified about the status of their papers by January 15.

7. The Colloquium is scheduled in April.

SUGGESTIONS TO THE AUTHORS

When preparing a research paper include:

a) a rationale and an identification of the research question(s)

b) a conceptual framework or brief statement of relationship to the literature

c) an identification of research methodology

d) a summary of the analytical technique(s)

e) a summary of preliminary findings

f) conclusions and discussion

SUBMIT PAPERS AND CORRESPONDENCE TO:

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The Role of Language in Childhood Cognitive Development and Reasoning: The Influence of the Work of Piaget and Vygotsky  
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Andrew Golay is a mathematics teacher at Lowell High School, Lowell, MA. He earned a Bachelor's degree in mathematics in 2003 and a Master's degree in education (curriculum and instruction) in 2004. He is a prospective Ed. D student at the Mathematics and Science Education doctoral program.

Peggy LaBrosse teaches chemistry at Hollis/Brookline High School in Hollis, NH. As a member of Northern New England Co-Mentoring Network, she mentors new high school science teachers. She is a doctoral student in the Mathematics and Science Education program and is currently working on her qualifying paper.

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2004-2005 Academic Year

Dissertation Defense Stage

Suellen Robinson
Volition and Its Relationship to Retention in Community College Mathematics Classes

Adele Miller
An Investigation of College Students' Rational Number Sense in the Developmental Mathematics Classroom

Jeff Todd
Middle School Mathematics Teachers' Use of Systematic Lesson Planning and Its Relationship to Student Mathematics Achievement in Low Performing Urban Schools

Dissertation Proposal Stage

Rocco Perla
The Development and Validation of a Case Study in Science in the Context of a Non-Linear Model of Scientific Change

Scott Stanley
A Study of the Issues Related to the Development of Web-based Science Courses

Kathy Shea
Mentoring Alternatively and Traditionally Licensed First Year Science Teachers

Qualifying Paper Stage

Danielle Cross
Marsha Pease
Jessica Devonis
Kate McLaughlin
Alex Ballantyne
Peggy LaBrosse
ABSTRACT

This paper describes the development of understanding, by the author, of the work of Vygotsky and Piaget in the context of the emergence of language in the child and its relationship to thought. This evolved throughout the Reasoning and Problem Solving in Mathematics and Science course I took in spring, 2003. Despite some differences in perspectives, the two philosopher-psychologists stress the importance of the role that language skills play in the development of the ability of the child to reason. The development of an "inner voice" or articulated, verbalized thought is a central theme in their seminal works on the development of higher order mental functions. Their different focus and perspectives on individual cognitive development and the role of cultural and social transfer of information, leading to the construction of knowledge, is examined synergistically. Both authors have had seminal influence on the understanding of cognitive development of the child. The orthogonality of, and holistic connections between, their work has had a significant synergistic influence on the educational community. The constructivist epistemological movement has borrowed heavily from understandings of their work in order to examine and understand how the individual constructs knowledge. This has had significant pedagogical implications and application of their ideas to the science classroom through both discovery learning and Socratic dialogue.

"The reason that cliches become cliches is that they are the hammers and screwdrivers in the toolbox of communication". (Pratchett, 2001)

This paper reflects the learning that I went through in taking a course in Reasoning and Problem Solving in Mathematics and Science at the Graduate School of Education during 2003. This follows a developing interest in the interactions between language, reasoning in both scientific and mathematical literacy and higher order thinking. Having had many years of experience in engineering research and the physical sciences, prior to undertaking my new avocation into the realm of secondary science education, it became apparent that the new perspectives are needed. The educational process in mathematics and science requires the educator to have a deep understanding of the role of communication in cognitive development. This fulfills two objectives: as a practitioner, it provides a greater awareness of my students needs, and secondly, a fulfillment of my intellectual curiosity as to a previously unrecognized and unappreciated role of language in cognitive development.

In consequence, this paper addresses the influence of language upon the development of thinking skills, in the context of the seminal works of Piaget and Vygotsky. This has a broad context for human cognitive development and is of particular importance to the understanding of reasoning in the scientific and mathematical disciplines. This paper examines the works of each of the authors in the light of their overall contributions to psychological theory. More specifically, it is primarily directed towards their contributions in the pursuit of understanding of the role that language, signs and symbols play in the tapestry of the burgeoning intellectual growth of the child. In particular the synergism between individual cognitive development and learning, and the socio-cultural transmission of, information, skills and ultimately, transformation into knowledge, is explored. In the following sections, the historical background and implications of the works of Piaget and Vygotsky are examined in the context of their lifework, as well as an in depth exploration of their work associated with language development in the child. As Pratchett (2001) humorously wrote in the above quotation, this paper examines the role of the toolbox of linguistic skills in human cognitive development.

PIAGET IN PERSPECTIVE

Jean Piaget was born in Neuchatel, Switzerland in 1896. He became a well-known expert on molluscs by the time he finished high school (Piaget Society, 2000). His interest in biology lead him to obtaining a Ph.D. in natural sciences at the University of Neuchatel at age 22. It was then that he developed an interest in psychoanalysis, moving to France and then back to Geneva. During the 1920's, he became very interested in cognitive psychology and the role of language in psychological development of the child The Language and Thought of the Child, (Piaget, 1926/1959), first published in French as Le langage et la Pensee Chez l'Enfant, (Piaget, 1923). His major interests and areas of research included, psychology of intelligence (Piaget, 1936/

Piaget had a long and very productive professional career, publishing over 50 books and 500 papers. He created the International Center for Genetic Epistemology in 1955, for which he remained director until his death in 1980. His legacy is one of the most significant in cognitive psychology. Smith (1992) wrote that, "Piaget was a philosopher's psychologist who displayed an exceptional interdisciplinary expertise in making a major contribution to human knowledge." His work did not find access and popularity in America until the 1960's, with the publication of The Developmental Psychology of Jean Piaget (Flavell, 1963). At present, his work has become central to the understanding of problem solving and reasoning in science and mathematics (Nickerson, 1986; Mayer, 1985, 1992). In particular it has become a central theme in the philosophical framework of constructivism (see, for example, Steffe and Gale, 1995).

The cornerstone of Piaget's evolving understanding of cognitive development is that of his genetic epistemology. His biological background persuaded him that the key element of the child's psychological development lay in the generation of logically embedded structures, which evolved sequentially into larger and more complex forms with adulthood. His biological orientation saw intelligence and cognitive development as "mental embryology" (Flavell, 1963). Thus intelligence bears a "biological imprint;" although our specific genetic inheritance of sensory and neurological structures facilitates intellectual function, they do not account for the functioning itself (Flavell, 1963). The defining attributes, or functional invariants, of intellectual function, which are biological in nature are organization and adaptation. Adaptation is constructed of two elements: assimilation is the filtering or modification of input, and accommodation is the modification of internal cognitive schemes to fit reality (Piaget & Inhelder, 1969). The act of intelligence where there is balance or equilibrium between assimilation and accommodation is that of intellectual adaptation.

The mental, or cognitive, conceptual structure, that Piaget proposed is known as the schema. This is the continuing organizational structure, which is modified to reflect changes in the individual's understanding of reality. These schemas are perceived to develop through four sequential stages: the first is the sensory-motor stage which occurs approximately up to two years of age, the preoperational stage between ages two and approximately seven years of age, the concrete opera-

VYGOTSKY: A SHORT LIFE

Lev Vygotsky is an enigma: we know little of his life as he left no memoirs (Kozulin, 1986a). He was born the same year as Piaget, in 1896, in the town of Orscha in Belorussia. While studying law at Moscow University, he was also enrolled at the Shaniavsky University majoring in history and philosophy. He was interested in theater, poetry and linguistics (Kozulin, 1986a). In this regard his educational and intellectual background parallels the wide and interdisciplinary interests of Piaget. His career began with his Ph.D. thesis entitled The Psychology of Art, presented in 1925 at the Moscow institute of Psychology. This work was not published in English until 1971. Kozulin (1986a) represents this work as being indicative that for Vygotsky, "culture and consciousness constituted the actual subject of inquiry, while psychology remained a conceptual tool, important but hardly universal." In this regard Kozulin perceives more generally that Vygotsky's work is centered on human functions as opposed to natural or biological ones. Vygotsky's perspective of the role of psychological research was that it should investigate the mechanisms, which differentiate human conduct from animal behavior (Kozulin, 1986b).

Alexander Luria influenced Vygotsky's career. Luria was academic secretary at the Moscow Institute of Psychology, who obtained for him a position as a research fellow at the institute. While at the institute, Vygotsky explored cognitive development, in a context of interrelation between higher and lower mental functions, while including socially meaningful activity as an explanatory principle. He took lower mental functions to include perception, memory, attention, and will. He saw higher mental functions to be cultural or specifically human ones, which emerge gradually as radical transformations of the lower ones.
Vygotsky's ideas were radical at the time, and his early death of tuberculosis in 1934, combined with conflict with Soviet political doctrine placed them in a state of limbo. Consequently, his works were not available in the Western World until the 1960's (Kozulin, 1986a). His seminal works, Thought and Language (Vygotsky, 1962/1986), and Mind and Society (Vygotsky, 1978), a collection of his writings, assembled into book form. In the introduction to Vygotsky and Education, Moll (1990) quotes Bruner (1987), who wrote in his Prologue to Vygotsky's Collected Works (Vygotsky, 1987), that

When I remarked a quarter of a century ago that Vygotsky's view of development was also a theory of education, I did not realize the half of it. In fact, his educational theory is a theory of cultural transmission as well as a theory of development. For 'education' implies for Vygotsky not only the development of the individual's potential, but the historical expression and growth of the human culture from which man springs. (p. 1)

The work of Piaget and Vygotsky show a strong parallel, in terms of the orthogonality of their theories, in the domain of cognitive development. While Piaget looked more closely at the development of the individual child, in terms of her own resources, Vygotsky looked more closely at the development of the individual in the domain of cognitive development. While Piaget (1959) uses the term, unconscious, as not being fully under the control of the child. The development of verbal reasoning, through development of Vygotsky's inner voice (words without speech), depends strongly on the concept of word meaning. Vygotsky suggests that word meaning is different from the intrinsic nature of meaning ascribed to by Gestalt and association psychology. He writes that a "word does not refer to a single object," but is related to a complex generalized structure: in Piaget's terms, a complex conceptual schema.

This generalized meaning will vary from individual to individual, based on their prior experiences.

In a mathematical problem solving context, Mayer (1992) describes problem representation as consisting of three elements: linguistic knowledge, which pertains to word meaning; semantic knowledge, which relates to factual information, and schematic knowledge, which relates to a knowledge of problem types. From a combination of these three types of knowledge, the problem can be translated and integrated into an internal mental representation. Lester (1980) examined the problem solving literature, finding that there were indicators that linguistic predictor variables in problem solving performance are age- or grade-related. The ability to solve problems clearly goes beyond the linguistic, but it provides a necessary part of structuring the initial mental representation of the problem.

**VYGOTSKY AND PIAGET ON LANGUAGE AND THINKING: DEVELOPMENTAL CONTEXT**

Piaget states that the first appearance of semiotic function arises after the very young child has formed and used signifiers which are perceptual and are not as yet differentiated from the signified (Piaget & Inhelder, 1969). Typically in the second year of life, the idea of delayed or deferred imitation occurs. This is followed by
symbolic play, in which the signifier (such as the ritual of going to bed) shows a clear representational process in which the signifier is an imitative gesture, accompanied by objects which are becoming symbolic. This is followed by an intermediate stage with drawing, which occurs after two-and-a-half years of age. Internalized imitation follows, in the form of mental image, leading finally to the occurrence of nascent language: this representation occurs for example when the child says "Anpa bye-bye" some time after grandpa went away (Piaget & Inhelder, 1969). This leads to the semiotic "Anpa bye-bye" some time after grandpa went away (Piaget & Inhelder, 1969). This leads to the semiotic function having two instruments: signs, which are arbitrary or conventional, and symbols, which are motivated and may be created by the child herself. Initially such symbols are individual creations, but later include collective representations; therefore, such collective symbols are social in context. At these early stages of the use of symbols, Piaget and Inhelder (1969) describe this as the "transition from prerepresentation in action to internal representation or thought."

Piaget and Inhelder (1969) view language as being derived genetically. He argues this in the comparison of the normal child who begins to use language at the same time as the other forms of semiotic thought, with that of the deaf-mute, for whom, articulate language is significantly delayed. Language makes its appearance at the end of the sensory motor stage in the form of one-word sentences. By the end of the second year two-word sentences appear, which are complete in themselves but lack grammatical structure. Transition to grammatical speech is significant in the context of linguistic and information theory, but Piaget raises concerns in regard to thought and logical operations. Specifically he recognizes the way in which language expands the range of thought in the young child, but not in the sense of logico-mathematical structures, as these may or may not be of a linguistic nature.

One of the significant aspects of Piaget's early work (Piaget, 1959) is that of his use of careful observation of the child in social settings. Detailed recording of the speech of children in such a setting led him to observe that a significant fraction of the articulated speech of the pre-operational child is egocentric. By this he means that the child "talks only to himself or for the pleasure of associating anyone who happens to be there with the activity of the moment" (Piaget, 1959). This egocentric speech therefore falls into two categories: monologue and collective monologue. Investigations of the extent of such speech in a class of six-year-old students gave a figure of 45% (average variation = 5%). This is balanced by a second component: social speech. Piaget points out that the young child appears to talk in a social context but a significant portion of this is really not true social speech. The child's talk is often associated with play, and the interaction between children often takes the form of threats, commands, and requests, which facilitate the actions within play. There is little in the way of reflection or the sharing of thoughts.

Piaget considers the evolution of the child's thought to that of the adult to require a change of perspective from self to that of the point of view of others. For children of 7 or 8, Piaget concludes that there is no real social life between them. Thus the development of understanding between children of this age is not a key factor: the effort to understand and to communicate thoughts objectively does not occur because of the child's egocentrism (Piaget, 1959). Piaget considers two modes of thought in the child: directed thought and autistic thought. Directed thought is controlled by experience and logic, while autistic thought is concerned with symbolism and immediate satisfaction. Thus, the intelligent mind regards an object in the light of its empirical behavior, and its physically determined characteristics. Directed thought is conscious and is intelligent in that it is adapted to reality and attempts to influence it. Autistic thought is centered on the purely organic character of the object: its intrinsic value in terms of wants and needs. Vygotsky (1986) sees a tendency of Piaget to emphasize the similarity between egocentric and autistic thought, although Piaget warns about this (1959). Vygotsky (1986) views the process of language and thought proceeding from a social speech, driven by interaction with the surrounding community of individuals, followed by egocentric and then inner speech. Piaget views thought as proceeding from autistic nonverbal thought, through egocentric speech to socialized speech. This distinction, in terms of the role of social speech may be a matter of definition of the term. Clearly, Piaget and Inhelder (1969) view language as a collective social phenomenon. By its nature, word meaning, as transferred to and constructed by the child, is a symbolic representation which is mediated through cultural and social contact. The differences in perspective are not as profound as Vygotsky would suggest. Language proceeds to the development of inner speech or articulated thought in both of their cognitive schemas.

Piaget examines the ideas of syncretic thought in children up to eleven years of age, where their interpretation of meaning is derived from the fusion of words into a projected meaning, which does not require reflection of analytic reasoning. This can lead to a generalized schema, which nonetheless contains misunderstandings. However, Piaget views this as a part of a
progressive adaptation, which eventually leads to full understanding. Thus Piaget sees language understandings during this developmental stage as being more related to autistic thought than to true logical thought.

Vygotsky had a different perspective on the process of development of thought and language. Piaget studied groups of children, in the context of interaction between members of the peer group. Vygotsky was conscious of the interaction between adult and child. This led him to a viewpoint that was in conflict with Piaget's concepts of childhood egocentrism. Vygotsky distinguished between spontaneous thought and scientific thinking. *Spontaneous thought* is associated with the child's everyday experience. *Scientific thought* is associated with systematic learning: in other words, with systematic instruction by a mentor adult. In this context he sees an educative process, which is by its very nature systematic instruction by a mentor adult. This led him to a viewpoint that was in conflict with Piaget's concepts of childhood egocentrism. Vygotsky distinguished between spontaneous thought and scientific thinking. *Spontaneous thought* is associated with the child's everyday experience. *Scientific thought* is associated with systematic learning: in other words, with systematic instruction by a mentor adult. In this context he sees an educative process, which is by its very nature social and cultural. His investigations of 2nd and 4th grade students show a significantly higher level of conscious comprehension of scientific concepts than spontaneous ones (Vygotsky, 1986). Because the scientific conceptual development of the child is socially defined, the mechanism for the child's increase in understanding must be driven by conscious construction of verbalized thought. This reflects itself in the concept of the development of an articulated inner speech which acts as a mediator for development of higher order mental function (1986). This conceptual structure is clearly related to Piaget's ideas of adaptation and transition to formal operations and abstract thinking. As pointed out in Piaget and Inhelder (1969), the development of the child towards adult formal operational thinking is not just a social or cultural one. It is important that the individual child internalizes information and experience into cognitive schemas. Although necessary and essential, it is insufficient in itself. Piaget and Inhelder (1969) state:

Socialization is a structuration to which the individual contributes as much as he receives from it, whence the interdependence and isomorphism of 'operation' and 'cooperation'. Even in the case of transmissions in which the subject appears most passive, such as school-teaching, social action is ineffective without an active assimilation by the child, which presupposes adequate operatory structures.

**SIGNIFICANCE OF LANGUAGE AND THOUGHT IN PEDAGOGY**

It is clear that the issue of language plays a major role in research into scientific and mathematics pedagogy. As an example of this connectivity with pedagogical research, the positions taken with regard to discovery learning (Hendrix, 1961), show strong links to the work of Piaget and Vygotsky. For example, a major point raised by Ausubel (1964) is that of his conflict with Hendrix's concept of *subverbal awareness* (Hendrix, 1961). The conflict centers on the value of verbalization of conceptual understanding. Hendrix's position is that the discovery process provides effective learning at a nonverbal level. This is a concrete-experimental level of understanding, which is intuitive, correlating with Piaget's pre-operational and concrete stages of development. In a sense, this is Piaget's autistic thought (1959). Ausubel's perspective is that this ignores the role of language in the cognitive development of higher levels of abstract thought. This raises Vygotsky's (1986) position on language and thought, which focuses on the development of higher order thinking as a result of development of an inner voice. In other words, developed and rational thought is, essentially, a sub-vocal verbalization. Once this perspective is accepted as providing a plausible model of the process of thinking, Ausubel's position becomes clear.

The development of complex cognitive structures and conceptual schemas cannot rely on purely intuitive understanding. Cultural transmission of ideas therefore must rely on the ability to communicate effectively. This requires superior verbal skills. Where do such skills develop? Vygotsky reduces this problem to one of reasoning through internal speech, and thus developing and expanding the levels of abstraction that the child is capable of attaining. From a Piagetian perspective (Piaget & Inhelder, 1969), creation of complex abstract conceptual schemas requires accommodation of incoming information into structured and well-organized patterns of understanding, through reasoning, internal dialogue and argument. From Ausubel's (1964) position this is stated in the context that verbalization "constitutes, rather, an integral part of the very process of abstraction itself." The verbal expression of an idea is therefore, not just the encoding of subverbal insight into words, but rather an expansion towards a much higher level of insight. As Ausubel states, this generates "clarity, precision, generality, and inclusiveness" which is not possible at the subverbal level. This begs the question as to the applicability of Hendrix's teaching methodology: Hendrix (1961) does not appear to...
specify the Piagetian developmental stages that are appropriate for subverbal awareness, but Ausubel posits a plausible scenario. An interpretation of subverbal versus articulated thought centers on the level of development in elementary age school children—pre-operational and concrete operational, where the levels of language development and reasoning are only beginning to find significant growth. Without fully developed language skills, the ability of the child to fully explore and understand complex concepts is constrained. Therefore, experiential-concrete and intuitive understanding is served by discovery learning. However, at the secondary level of education the transition to Piaget's formal operational thought allows the student to make a transition to expository learning, such as is effected by Socratic dialogue. The key issue to such a transition is the need for the child to have developed such language skills as are needed for formal operational thought. Without use of some form of verbal thinking and dialogue in the elementary school environment, the child's potential for development as an abstract thinker will be limited.

Thus Ausubel's (1964) statement of "clarity, precision, generality, and inclusiveness" in language epitomizes Piaget's and Vygotsky's perspectives and becomes a significant driver in the pedagogical imperatives that concern the teacher of science and mathematics.

CONCLUSION

This study of the works of Piaget and Vygotsky has been a profound learning experience for me. A tremendous shift in my perspectives has occurred as a result of this study. Both authors have had seminal influence on the understanding of cognitive development of the child. The orthogonality of their work, contrasting individual versus cultural and social influences, had a significant synergistic influence on the educational community. It is the holistic connection between the works of these two authors that magnifies their impact, generality, and inclusiveness in language epistemic activities. An interpretation of subverbal awareness, but Ausubel posits a plausible scenario. An interpretation of subverbal versus articulated thought centers on the level of development in elementary age school children—pre-operational and concrete operational, where the levels of language development and reasoning are only beginning to find significant growth. Without fully developed language skills, the ability of the child to fully explore and understand complex concepts is constrained. Therefore, experiential-concrete and intuitive understanding is served by discovery learning. However, at the secondary level of education the transition to Piaget's formal operational thought allows the student to make a transition to expository learning, such as is effected by Socratic dialogue. The key issue to such a transition is the need for the child to have developed such language skills as are needed for formal operational thought. Without use of some form of verbal thinking and dialogue in the elementary school environment, the child's potential for development as an abstract thinker will be limited.

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Educators are applying their ideas to a new perception of the educative process and they will increasingly influence pedagogy. The example of discovery learning illustrates the importance of relating Piaget's and Vygotsky's concepts of internal articulation of verbal thought to the development of higher order mental functions. As a teacher of secondary science, my emerging awareness of the need for a clear, precise, and higher level articulation of language, as a fundamental and necessary skill, has influenced my perception of the directions explicit in my future teaching practice.

References


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*Science Education*

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ABSTRACT
This paper examines whether the sequence of high school science courses — physical science, biology, chemistry, and physics — matters in promoting student understanding of science concepts and achieving scientific literacy. In the present sequence of science courses, ninth graders take physical science, tenth graders take biology, eleventh graders take chemistry, and twelfth graders take physics. Science electives include environmental science, anatomy and physiology, and astronomy. However, the core science courses — physical science, biology, chemistry, and physics — are the focus of this paper. The sequence of science courses — biology, chemistry, and physics — was set out in 1894 by the Committee on Ten (Lederman, 1998). According to Lederman, the sequence is inappropriate and does not respect developments in the disciplines over the past century. Lederman (1998) presents a model for teaching high school sciences based on a set of principles “coherence, integration of the sciences, movement from concrete ideas to abstract ones, inquiry, connection and application, sequencing that is responsive to how people learn” (p. 1). Scientific literacy for the 21st century is defined, the concepts necessary for scientific literacy are outlined, and the content standards necessary for biology are presented. Recommendations for the sequence of courses are made, and alternative suggestions for increasing the number of science students can take are included.

INTRODUCTION
The terms and circumstances of human existence can be expected to change radically during the next human life span. Science, mathematics and technology will be at the center of that change — causing it, shaping it, responding to it. Therefore, they will be essential to the education of today’s children for tomorrow’s world.

What should the substance and character of such education be?

(American Association for the Advancement of Science, 1993, p. XI)

Several eighth-grade students’ families have questioned whether their children can take Honors Biology as ninth graders. The reason for this request is to allow more room in their schedules in tenth through twelfth grade to take additional advanced science courses. This seems a simple request on the surface — provided there is space available in the classroom, enough textbooks and materials, and their schedules will accommodate the class, why not? However, on closer examination, it behooves us as educators to ask two questions before making a decision:

• First, “Is this in the student’s best interest?”
• Second, “What data and research support the decision?”

This research paper proposes to answer both questions based on a review of existing literature and interviews with educators who are using best practices. In this case, best practices refer to educational practices based on the student’s best interest, data, and research.

This paper comprises several sections. The first section summarizes conversations with educators presently involved in research and best practice in science. The next section outlines some of the current research and defines what it means to be scientifically literate in the 21st century. The final section includes a summary and some recommendations. This research is by no means all-inclusive; rather, it is an attempt to gather a cross section of information for making an initial decision and provide recommendations for further study. Each school district is unique. Any implementation of these recommendations accommodate that uniqueness, keeping in mind what is best for the students based on existing research and data.

CONVERSATIONS WITH PROFESSIONALS
Several e-mail exchanges with educators who actively use best practices, based on data and research, helped to focus this research. The e-mail exchanges with Joyce Tugel (2004), from The Education Resource Center (TERC), and Susan Mundry (2004), from West Ed, centered around the concepts that students need for biology, an example of a local school that teaches biology in the ninth grade, and an example of a school that
teaches physical science in the ninth grade, along with the results in each case. The consensus expressed in these e-mail exchanges is that physical science concepts are needed for understanding biology. One school that has been successfully teaching Physics First was cited as an example (Mundry, 2004):

Farmington High School in Connecticut, where Physics First has been in place for ten years:

- 100% of students complete a full-year physics course
- The number of freshmen enrolled at the honors level has more than doubled
- Enrollment in AP science courses has more than tripled
- Freshman taking the SAT II Physics Achievement Test (2003) had a mean score of 692

A face-to-face conversation with Barbara Hopkins (2004) included a sincere warning to avoid moving biology into the ninth grade. Concord High School (in Concord, NH) did so several years ago and is now trying to fix a problem where students are not doing well at learning biology. The fix is costly in time, money, and student achievement. Barbara (2004) is involved with remediation at this point, and her experience is well worth attention.


From our conversations, the importance of a background in physical science for understanding biology concepts emerged. The following research should help inform the decision about the sequence of high school science courses.

A BRIEF SURVEY OF EXISTING LITERATURE

The work of the Biological Sciences Curriculum Study (BSCS, 2004) has been ongoing since 1958. The leadership of BSCS has benefited science education in various ways. For example, BSCS has placed inquiry into the curriculum and provided models for its use by science teachers. Perhaps even more important is the introduction of connections between science and society, leading to scientific literacy (BCBS, 2004). However, even with these innovations, we apparently are still falling short in producing scientifically literate citizens. In BSCS (2004), a keynote address by Leon Lederman provides his observations:

...If we now focus, not on future physicists, or even on future scientists, but on future citizens, then we find that the public science literacy, that is the level of understanding of science by the populations of many nations, developed and developing, is far from what a 21st-century world requires.

This indicates that in much of the world, education to approximately age 18 does not produce science-literate citizens who can participate in the decisions of their communities and nations as to how to use the vast potentialities of 21st-century science and technology. These educated citizens can also participate in decisions as to which technologies are beneficial to the long-term future of nations and which may have adverse effects. In contrast, all measurements of scientific literacy find Americans woefully ignorant of what science is, what it can do, and what it cannot do. The popularity of “junk” science, of astrology, of UFO enthusiasts, of wildly alternative medicines, of fortune-tellers and creation “science” adherents bespeaks a population ill-fitted to use their democratic prerogative to contribute to 21st-century global problems such as AIDS, environmental degradation, overpopulation, and vanishing natural resources. (p. 10)

In 1998, Leon Lederman proposed that the traditional biology–chemistry–physics sequence of science courses in high school was outdated (1998). Lederman's project advocates a revised physics–chemistry–biology sequence as one that better accommodates advances in science over the past century, as well as student needs. Lederman (1998) supports the need for physical science concepts with this example:

A key biological content standard is “matter, energy and organization in living systems.” All energy used by living systems ultimately comes from the sun through electromagnetism (light). Energy transformations can be explained using concepts from chemistry. Through energy transformations, plants make food (energy) which flows through the ecosystem. The availability of energy largely determines the distribution of populations (organisms) in the
ecosystem. Atomic and molecular reactions with photons (light) and with one another are the underlying phenomena. (p. 18)

This example is further supported by a key finding described in How People Learn (National Research Council, 2000).

To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge [student understandings from physics about atoms and molecules], (b) understand facts and ideas in the context of a conceptual framework [student recognition of energy transformations through hierarchies of living systems—plants–populations–ecosystems], and (c) organize knowledge in ways that facilitate retrieval and application [student understandings of relationships among concepts across the scientific disciplines]. (p. 16)

The final evidence for the importance of physical science preceding biology is presented in Table 1. As shown in the table, some concepts from each of the six content domains specified in the life science standards for grades 9-12 draw upon understandings from the 9-12 physical science content standards.

### Table 1. Dependence of Life Science Content Standards on Physical Science Content Standards

*Biological Science Curriculum Study, 2004, p. 3*

<table>
<thead>
<tr>
<th>Life Science Content Standard</th>
<th>Physical Science Content Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Cell</td>
<td></td>
</tr>
<tr>
<td>Cells contain a mixture of thousands of different molecules that form the cell membrane and other cell structures.</td>
<td>![checkmark]</td>
</tr>
<tr>
<td>Most cell functions involve chemical reactions (such as the break down of molecules to store energy in other chemicals and the synthesis of molecules using the stored energy).</td>
<td>![checkmark]</td>
</tr>
<tr>
<td>Plants and many microorganisms use solar energy to build organic compound, using CO₂ from and releasing O₂ to the environment.</td>
<td>![checkmark]</td>
</tr>
<tr>
<td>Molecular Basis of Heredity</td>
<td></td>
</tr>
<tr>
<td>DNA is a large polymer formed of four subunits, adenine (A), guanine (G), cytosine (C), and thymine (T).</td>
<td>![checkmark]</td>
</tr>
<tr>
<td>Biological Evolution</td>
<td></td>
</tr>
<tr>
<td>Natural selection and evolution provide a scientific explanation for similarities observed among molecules in diverse species.</td>
<td>![checkmark]</td>
</tr>
</tbody>
</table>

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CONCLUSIONS AND RECOMMENDATIONS

The research (American Association for the Advancement of Science, 1989; 1993; Lederman, 1998; National Research Council, 1996; 2000) supports the notion that students must first be presented the concepts of physical science to understand biology. Therefore, if we are to base decisions on research and what is best for students, it would seem that the concepts of physical science should precede a course in high school biology. The present curriculum at Hollis/Brookline High School (HBHS) for seventh-, and eighth-grade science presently does not contain any of the physical science concepts needed for student understanding in biology. Seventh-grade science is health- and life-science oriented and eighth-grade science is earth science. The ninth-grade curriculum includes the physical science content standards listed in Table 1. The present sequence of courses at HBHS is supported by research that puts student needs first.

So what kind of science education is needed to meet the challenges of our rapidly changing world? A world where new discoveries are occurring at an exponential rate requires that a scientifically literate person have the habits of mind that allow him or her to ask questions answerable using a logical process based on data collection, synthesis, and analysis. People need to be able to make sense for themselves; it is not possible to construct knowledge as fast as technological change is creating it. Did you ever think there would be so many options for fertility therapy? How about knowing the sex of a baby before it was born and taking pictures of it? These are just a few emerging changes. Stem cell research is a huge field, and just knowing the parts of a cell does not come close to helping someone make an informed decision about the legislation related to this topic, yet it is a contemporary issue. My point is that it is our responsibility as educators to prepare students to become scientifically literate citizens.
The science education research community (American Association for the Advancement of Science, 1989; 1993; Lederman, 1998; National Research Council, 1996; 2000) has only begun to scratch the surface about how students learn, what understanding means, how knowledge is synthesized, and how to be an independent thinker. Project 2061 (American Association for the Advancement of Science, 1993), the National Research Council (National Research Council, 1996; 2000) and Driver (1994) have done a lot of research into the developmental appropriateness of concepts. In addition, Ausubel (Ausubel, 1978) researched how students develop concepts. From Ausubel’s (1978) research, it is clear that students are ready for different concepts at different developmental stages, and are able to build these concepts into meaningful structures when pre-existing structures are first put into place. In other words, there is a logical sequence to knowledge building.

So what about ninth graders taking biology? Several area schools, Bishop Guertin and Concord High School (Clare, 2004; Hopkins, 2004) in particular, have put this into practice and the students have not performed as well on the tenth-grade New Hampshire Educational Improvement Assessment Plan (NHIEAP) testing. In both of these schools, teachers are noticing that students are not prepared for certain biology concepts. As a result, the teachers have had to choose between removing those concepts from the curriculum, which adversely affects the tenth-grade testing, or keeping the concepts in the curriculum but having to grade on a curve (Voth-Palisi, 2004). Both choices are unsatisfactory. The students need the physical science concepts prior to biology. In both schools cited, physical science did not precede biology. Concord High School has hired several experts, including Barbara Hopkins and Bruce Wellman (Hopkins, 2004; Lipton, Wellman, & Hubbard, 2003) to provide a solution. At this point, they are considering merging physical science and biology into a two-year course in “integrated science.” While this would allow physical science and biology concepts to be taught together, as appropriate, it has several drawbacks. First, it is expensive. In addition, colleges are not accustomed to “integrated science.” Instead, most colleges look for transcripts that reflect a traditional biology, chemistry, physics track.

The students at HBHS achieved the top tenth-grade science scores during NHIEAP testing in 2002-3. The numbers of students accepted to 4-year College is exceptional. Admirably, HBHS parents want more for their children. So do the people at School Administrative Unit (SAU) 41. Our students are our future, and we cannot afford to make cavalier decisions based on vague notions, desires, politics, or other subjective criteria. Instead, major decisions affecting student learning need to be data driven and research based (Hayes-Jacobs, 2004). With that in mind, the leadership of SAU 41 asked a member of the science curriculum task force—Peggy LaBrosse—to spend the summer researching, interviewing educators who exemplify best practices, and writing this paper. The goal that students take more advanced science can be met in several ways:

- Physical science can be offered during a summer session preceding the ninth grade.
- Motivated eighth graders can be allowed to take ninth-grade science at HBHS during their eighth-grade year, in addition to the recommended course load.
- Students can double up on science in the later years.
- Physical science concepts can be included in the middle school curriculum.

We are certain that this goal can be met in a variety of different ways. However, at this time, it would be inappropriate to make a change without first considering the available data and research. At present, science concepts are offered at HBHS in a developmentally appropriate sequence, according to science education research and NHIEAP test results. In addition, more Advanced Placement (AP) and advanced science courses are being added to the high school curriculum.

Any social-science research is fraught with uncontrolled variables. There will be exceptions to what is written here and reported in the research. There are those precocious students who are gifted to be able to make connections, and build the knowledge even when it is presented in a less than prescribed manner. That is what makes this field exciting and rich. However, if — as the Annenberg Private Universe (Schneps & Sadler, 2003) portrays — Massachusetts Institute of Technology (MIT) graduates cannot explain where the matter in a tree comes from, and they are supposedly the best of the best, then what does that have to say about how well we are doing? Without clear knowledge one cannot really understand cycles in nature, photosynthesis, global warming, and this basic application of the law of conservation of matter.

Please take time to read some of the resources cited; these are the voices of the people who have been in the trenches, with the focus on only one person, that student of yours.
General Description of the Educational Doctorate in Mathematics and Science Education at UMass Lowell

Foundations .................. 6 credits
Research Courses .............. 9 credits (minimum)
Core Program Courses ........ 15 credits
Electives ........................ 6 credits (minimum)
Dissertation Research .......... 12 credits (minimum)

Students must pass a comprehensive and a qualifying exam prior to dissertation research.

Who Should Apply?

• K–12 teachers or Higher Education faculty who hold the equivalent of an undergraduate degree in mathematics, science, or engineering, together with a masters in any field.

• Teachers without an undergraduate degree in mathematics, science, or engineering, but who are PALMS specialists or National Board Certified in Science or Mathematics and hold a masters degree in any field.
ABSTRACT
This article extends and elaborates on our previous work published in volume VIII and presented at last year’s Colloquium, which addressed the distinctions between two antagonistic philosophical traditions known as logical positivism and postpositivism. The primary aim of that paper was to develop qualitative mathematical models of positivism and postpositivism based on clearly defined assumptions of each of these diametrically opposed philosophical traditions. During our presentation, and in subsequent discussions with participants of the presentation, it became clear that a number of concepts central to the viability of the mathematical models were imprecise and difficult to understand. Most notably, the definition of diffeomorphism and the “mapping” example were rather cumbersome. We resolved the issues by replacing the original definition of diffeomorphism with a simpler yet equivalent (more parsimonious) definition. The current definition is not only more in tune with the intuitive notion of a diffeomorphism, but it also made it possible to prove that the mathematical models constructed for positivism and postpositivism (which are certainly not the only possible representations of these traditions) are not qualitatively equivalent. In this article we provide the first mathematical proof of the popular scholarly conjecture that models of positivism and postpositivism are qualitatively different. We argue that rigorous mathematical models can be used profitably in comparative epistemology and should be considered by researchers in science education, since the nature of science is most often associated with the epistemology of science, or the values and assumptions inherent in the development of scientific knowledge. Furthermore, considering the ambiguity of language and the complexity and difficulty associated with philosophical and epistemological issues, it appears reasonable to use the precise language and structure of qualitative mathematics as a guiding framework, cognitive aide or scaffolding device for some learners in some learning environments. These types of instructional issues are addressed toward the end of the article.

INTRODUCTION
For the first half of the twentieth century logical positivism (or logical empiricism) was the dominant philosophy of science in the Anglo-American world and its influence on science education is well described (Duschl, 1985, 1994; Matthews, 2004). Although accounts and comparisons of logical positivism and postpositivism are common in the literature today, this subject has primarily been the jurisdiction of philosophers, historians, and sociologists of science, as well as science educators with a penchant for philosophical discourse. Conspicuous in its absence is a mathematical analysis and treatment of the epistemological features of these antagonistic philosophical traditions. The purpose of this paper, therefore, is to describe the qualitative mathematical distinctions between the two schools of thought using order and topology (Isnard & Zeeman, 1977). The relatively new development of a qualitative mathematical language (Thom, 1975) allows for the use of precise mathematical terms, concepts, and models, which can be used to describe a wide range of phenomena previously considered beyond formal mathematical treatment.

It should be emphasized that the views of positivism and postpositivism that will be described here are vulgarized versions intended only to address the basic epistemic assumptions of each program as they relate to theory change and progress in science. It is the main intent of this paper to model these basic assumptions using qualitative mathematics and not to provide an exhaustive or comprehensive review of each program. Although we recognize and appreciate the recent interest by the science education community to develop a more thoughtful and responsible appraisal of the positivist movement and viewpoints (see Science & Education, 2004, volume 13, issue 1-2), we believe that a “standard” model of positivism (or postpositivism) does not exist. Instead, we agree with Lincoln and Guba (1985) who make the following assessment based on a number of scholarly reviews of the positivist thesis:

Positivism can be reshaped, apparently, to suit the definer’s purpose, and while there is certainly remarkable overlap in these statements [of the commentators] there are also some inconsistencies and idiosyncracies. One might venture to say that the particular form of definition offered by a commentator depends heavily upon the counterpoints he or she wishes to make. (p. 24)
We certainly share these concerns and would add that the notion of epistemic change and stability is currently under scrutiny by philosophers looking to clarify these critically important terms and concepts (e.g., Gillies, 2004; Hansson & Helgesson, 2003). To the extent that the stability of knowledge claims plays a central role in nature of science studies, it is imperative that the terms and concepts used to express the stability and tentativeness of scientific knowledge be as clear and rigorous as possible, even if there is disagreement as to which models of scientific development, change and progress are most justifiable. The precise language of mathematics is ideally suited for such a task.

It is against this backdrop that the current research takes form. By recognizing important terms used to describe the nature of science, scientific knowledge and theory change by both positivists and postpositivists and then translating these terms into their more precise mathematical equivalents (where possible), much stands to be learned about each tradition by reducing conceptual ambiguity. We agree with Khait (in press) that “mathematics is an essentially linguistic activity characterized by association of words with precise meanings” (p. 1). Of course this precision creates problems of its own, namely the elimination of the excess meaning that is recognized as the creative and generative force in any intellectual field. However, this concern and issue is vitiated in the fact that an understanding of basic concepts and terms is believed to be an important consideration during the initial stages of learning and quite necessary for the development of more abstract thinking and conceptualization (Ausubel, 1968; Bruner, 1977). It is our hope that the ideas developed in this paper will engender a lively dialogue among science educators (and perhaps even between science and mathematics educators) who take a particular interest in the philosophical, historical, and epistemological dimensions of their respective field of study.

**AN OVERVIEW OF POSITIVISM AND POSTPOSITIVISM**

Logical positivism is a philosophical movement that developed in Austria and Germany during the 1920s under the influence and guidance of the Vienna Circle, a group of scholars that included philosophers, scientists, and mathematicians. The aim of the early positivist movement was strikingly lucid and extremely ambitious: to develop a foundationist account of scientific knowledge that would logically justify the scientific enterprise (Giere, 1988). As the name implies, foundationism is the view that all knowledge is based on a set of basic beliefs or “first principles” that are “self-evident.” Anything that is self-evident can be directly observed and perceived by the senses (e.g., the movement of celestial bodies). For positivists, experience and observation provided the foundation for all scientific knowledge, because the validity of observation claims could be determined empirically. Accordingly, positivists completely rejected all metaphysical claims or claims that were not empirically testable assertions of reality (Feigl, 1969). For example, assertions like, “God created the universe,” and “All knowledge is infinite.”
cannot be directly determined by logical or empirical analysis and are therefore meaningless.

In developing their own philosophy of science, positivists described scientific theories as axiomatic systems. These systems acquire an empirical interpretation through rules of correspondence. Rules of correspondence are analytic statements that link the abstract concepts of theories to experience and observation. In physics, for example, a correspondence rule might say that the symbol E in a formal axiomatic system is represented by the amount of energy released or absorbed by an atom. Here the symbol E, which is an abstract concept, is mapped to something that is empirically verifiable (i.e., the energy level of the atom).

Inasmuch as these analytical statements or “observational terms” were believed to be objective and theory independent, positivists argued that such terms were capable of determining the “true” meaning of a theory. In other words, theories do not influence the meaning of observations; rather observations influence the meaning of theories (Shapere, 1969). To the extent that observation terms were considered objective, knowledge claims took on an accretive or cumulative function. Indeed, logical positivism was a movement that from its earliest charter looked to construct a new philosophy of science, one that would provide the corpus of science with “epistemological guarantees” vis-à-vis formal axiomatic systems (Toulmin, 1969).

Although some of the concepts developed by the positivists still play an important role in science studies today (e.g., the importance of a precise scientific language), much of their program has been abandoned in favor of a less deterministic or dynamic interpretation of scientific knowledge. In part, this less deterministic approach argues that scientific theories are fallible (and therefore tentative) and that observations are influenced by existing theories. In arguing the latter point, Hanson (as cited in Eldredge & Gould, 1972) contends that “much recent philosophy of science has been dedicated to disclosing that a ‘given’ or a ‘pure’ observation language is a myth-eaten fabric of philosophical fiction...In any observation statement the cloven hoof-print of theory can readily be detected” (p. 85). This view of scientific knowledge is often associated with the ideas and theories advanced in Thomas Kuhn’s (1962/1996) classic book The Structure of Scientific Revolutions.

Based on a historical analysis of the physical sciences, Kuhn argued that scientists (a) see the world (and hence make observations) based on their guiding commitments, (b) compete vigorously for acceptance against competing and incommensurable views, (c) may experience a sudden or gestalt-like “switch” in theoretical commitment, and (d) may over time revert to a previously “rejected” idea or commitment based on “newer” findings, or new ways of looking at previously discarded ideas. These ideas are part of a dynamic view of the development of scientific knowledge, which sharply contrast the early positivist thesis. It is important to note that the ideas and theories explicated by Kuhn do not represent a de novo synthesis. Rather, they represent a point of imbrication among a number of postpositivist philosophers of science and science educators that predate Kuhn’s Structure (e.g., Fleck, 1935/1979; Hanson, 1958; Schwab, 1962; Toulmin, 1953). Kuhn’s work also has influenced and is consistent with more contemporary views in the philosophy of science (Lakatos, 1970; Laudan, 1977). It is this dynamic view of the development of knowledge, which we refer to as postpositivism. Using the precise concepts and language of qualitative mathematics, some of the similarities and differences between these two philosophical perspectives about the nature of scientific knowledge can be described in detail.

**WHAT IS QUALITATIVE MATHEMATICS?**

As Isnard and Zeeman (1977) point out, mathematics has three levels or types of structure: order, topological, and algebraic. Order structure involves ordinal-level measurements, which satisfy the transitivity postulate: If x is greater than y, and y is greater than z, then x is greater than z. Comparisons like “greater than” and “less than” are order concepts. Topological structure is measured using a continuous scale. The assumption of continuity allows for the moderated and careful use of some calculus concepts, such as inverse functions and differentiability. An inverse function is the function, f⁻¹(x), that “undoes” a function f(x). For instance, if f(x) = x + 2, f⁻¹(x) = x - 2. For an inverse function to exist for a particular function, the function must be one-to-one. A function is one-to-one if for every x there is no more than one y and for every y there is no more than one x. Differentiable means that a function or relation is continuous (it can be drawn without picking up your pencil) and does not have any hard corners, cusps or vertical tangents.

Algebraic structure involves operations such as addition and multiplication, which have little value in measuring sociological phenomena. As Isnard and Zeeman (1977) point out in their sociological model of the causes and tolls of war, “one cannot ‘add’ two senses of threat to get a third, and even twice the cost can become meaningless if one tries to include in the cost of
a war the measure of human suffering” (p. 321). Hence properties involving order structure and topological structure are defined as \textit{qualitative}, while properties that rely on algebraic structure are defined as \textit{quantitative}. This distinction is critically important because it implies that as long as changes between different scales are “smooth and order-preserving” (Isnard & Zeeman, 1977, p. 321) they are considered qualitatively similar, even if the algebraic structure is not preserved. This qualitative perspective allows for non-linear variations of scale, which are the type of variations that are recoverable from the theses of postpositivist philosophers of science such as Kuhn, Laudan, and Lakatos, among others.

\textbf{RECOVERING MATHEMATICAL MODELS FOR POSITIVISM AND POSTPOSITIVISM}

\textbf{A RECOVERY OF POSSIBLE MODELS FOR POSITIVISM}

Based on the brief overview of the early positivist thesis described above, we can, in hindsight, characterize this view of the nature of science as essentially accretive with respect to the development of scientific knowledge over time. This is in fact what Toulmin (1969) is implying when he refers to “epistemological guarantees” in science. Similarly, Losee (2004) has described this variety of progress in science as “progress as incorporation,” which emphasizes the gradual and incremental developments in science. Based on this view and interpretation of positivism, we can glean some basic mathematical assumptions that will help us to construct a qualitative model of positivism. Specifically, we focus on translating words such as “accretive” and “incorporation”—which are fairly representative of the positivist view of theory change and progress in science—to their mathematical equivalent expression, which can be translated to mathematics more precisely as: \textit{monotonically increasing over time}. It should be noted that the terms accretion and incorporation (and other related terms) in a mathematical sense do not assume or imply linearity, only continuity. This is because nonlinear models (such as those associated with quadratic, exponential and logarithmic functions on \( t > 0 \)) also increase monotonically. Also, to the extent that positivists describe how existing knowledge systems and structures develop over time to become more effective, their program meets the definition of a dynamic system, which represents continuous phenomena marked by change, activity and progress at varying levels of intensity and force. Therefore, from this description of positivism, we can glean some basic mathematical assumptions that will help us to construct a qualitative model of positivism. These assumptions are:

a) The acceptance of a fundamental scientific theory or group of theories is always monotonically increasing over time (the derivative with respect to time \( \frac{d}{dt} \) is positive for all \( t \)).

b) The development of scientific theories is continuous over time and represents a dynamic system.

Based on these assumptions, it is possible to construct many working models for positivism, four of which (linear, quadratic, exponential and logarithmic development) are shown in Figure 1.

\textbf{A RECOVERY OF POSSIBLE MODELS FOR POSTPOSITIVISM}

Because early positivists embraced a foundationist epistemology, they believed that established scientific knowledge was in many ways indubitable. Conversely, in examining the history of science, a number of scholars recognized that many of the important scientific achievements that provided the foundational edifice to a particular discipline have been rejected over time in favor of completely incompatible views. Indeed, this point is a central and recurrent theme among a number

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure1.png}
\caption{Various models of theoretical development in science under our assumptions of logical positivism.}
\end{figure}
of postpositivist philosophers of science and science educators. For example, in describing the transition in scientific thinking of infectious diseases from the mystical and fanciful to the etiological, Fleck (1935/1979) comments that “many theories pass through two periods: a classical one during which everything is in striking agreement, followed by a second period during which the exceptions begin to come to the fore” (p. 9). Eventually, as Fleck documents, these exceptions may lead to the rejection of previously venerated scientific commitments in favor of incompatible commitments and views. Similarly, Toulmin (1953) argues:

Now and then there may have to be second thoughts about matters which had been thought to be settled, but when this happens and the lower courses have to be altered, the superstructure has to be knocked down, too, and a batch of concepts in terms of which the scientist’s working problems used to be stated—‘phlogiston’ and the like—will be swept into the pages of the history books. (p. 81)

On the educational front, Schwab’s (1962) distinction between “stable enquiry” and “fluid enquiry” in science was intended to ameliorate the stagnant and immutable image of scientific knowledge proffered by most textbooks and science teachers at the time. The aim of stable enquiry is to contribute to and support an existing body of knowledge within a particular discipline. When extreme difficulties arise during periods of stable enquiry, fluid enquiry allows for the “development of new principles which will redefine that subject matter and guide a new course of effective, stable enquiries” (Schwab, 1962, p. 17). And Kuhn’s (1962/1996) “normal” and “revolutionary” science highlights periods of theoretical stability punctuated by relatively rare periods of instability and subsequent change in the physical sciences.

Of course these brief recapitulations do not do justice to the positions developed by each scholar, but simply serve to provide a flavor for the generally nonaccretive view of science that is the hallmark of postpositivism. In a scientific context, terms and expressions such as “nonaccretive,” “revolutionary,” “gestalt switch,” “complete rejection of a past theory,” and “a change in world-views” can be translated and reduced to the mathematical equivalent expression: not always increasing monotonically over time. Accordingly, we can recover two important mathematical assumptions from the ideas and theories explicated by the postpositivist philosophers of science:

a) The acceptance of a fundamental scientific theory or group of theories (i.e., a paradigm) is not always monotonically increasing over time \([t]\) (the derivative with respect to time is negative for some time interval \(t_1 < t < t_2\)).

b) The development of scientific theories is continuous over time and represents a dynamic system.

Based on these assumptions, it is possible to construct a model for postpositivism, one of which (a cubic curve) is shown in Figure 2. From a mathematics perspective, the postpositivist model is far more robust since it returns to its initial state of increasing acceptance over time. Also, it is possible that any positivist model is merely a “nonrevolutionary” section or segment of the postpositivist model. That is, once we admit that scientific knowledge is subject to changes in acceptance that increase non-monotonically, we can never exclude the possibility that this type of change will occur in a system where monotonic increases have been the rule. Juxtaposition of each of these models suggests that the primary difference between positivism and postpositivism is the assumption that acceptance is monotonically increasing over time (or that the rate of change is positive for all \(t\)). Again, from a mathematics perspective, it is also evident that both schools of thought are dynamic and continuous. This supports the contention of most philosophers of science who generally believe that regardless of the variety of change in science one supports or advocates (i.e., accretive models or non-accretive models) science is an enterprise that is progressive in nature (Losee, 2004).

Figure 2. A possible model of theoretical development in science under our assumptions of postpositivism.
EQUIVALENCE GRAPHS AND QUALITATIVELY EQUIVALENT MODELS

The aim of this section of the paper is to describe and compare the positivist and postpositivist models developed earlier using the concepts and language of qualitative mathematics. Because this language may involve a number of unfamiliar terms such as mapping, diffeomorphism, and qualitative equivalence, we provide a brief introduction and explanation of these terms and concepts.

**Mapping**

One may map a graph to a new graph using a transform, which is a rule that assigns each point on a graph to a point on a new graph. For example, the points that form the curve shown in Figure 3a could be mapped to figure 3b using the transform function \( g(x) = x + 2 \). Figures 3c, 3d and 3e show mapping functions of \( 3x \), \( |x-4| \) and \((x-8)^2\) respectively. Although each of the graphs in Figure 3 is a mapping or transform of the curve shown in Figure 3a, we need to establish specific mathematical criteria to determine the degree to which each of these graphs is equivalent to one another. It will be recalled that a change in scale between different graphs means that the graphs cannot be quantitatively equivalent. And in each of these graphs (Figures 3a-3e) there is a change in scale, so these graphs are not quantitatively equivalent. However, based on Isnard & Zeeman’s (1977) definition of qualitative equivalence, multiple graphs can sometimes have different scales, but still be considered qualitatively equivalent. The criteria for qualitative equivalence are described in the next section.

**Diffeomorphisms and Qualitative Equivalence**

The definition Isnard & Zeeman (1979) provide for qualitatively equivalent states that as long as a first graph can be mapped to a second graph using a diffeomorphism, then the graphs are considered qualitatively equivalent. A diffeomorphism is a mapping which is differentiable and has a differentiable inverse. The graphs in Figure 3b and 3c have been transformed from graph 3a using a mapping that is differentiable and has a differential inverse. In other words, a diffeomorphism exists that qualitatively maps these graphs to one another. Conversely, the transform used to map graph 3a to graph 3d is not a diffeomorphism, because the transform function \((x-8)^2\) is not one-to-one and therefore does not have an inverse function. Also, the transform used to map graph 3a to 3e is not a diffeomorphism because it is not invertible, nor is it differentiable at \( x=4 \). Thus, graphs a, b, and c are said to be qualitatively equivalent, whereas graphs d and e are not qualitatively equivalent to graphs a, b, and c.

**Mathematical Considerations of the Working Models of Positivism and Postpositivism**

Using the concepts and terms in the mapping example above, we can describe all of the positivist models shown in Figure 1 as qualitatively similar. That is, there exists a diffeomorphism that maps these graphs to one another. Using the diffeomorphisms \( x^2 \), \( e^x \) and \( \ln(x) \), the linear graph in Figure 1a can be transformed to the quadratic, exponential and logarithmic graphs shown in Figures 1b, 1c and 1d respectively. Again, since each of these transformations is differentiable with a differential inverse on \( t > 0 \), each of the graphs is qualitatively equivalent to the line shown in Figure 1a.

The criteria above also can be used to show that the positivist and postpositivist models shown in Figures 1a and 2 are not qualitatively equivalent. That is, no diffeomorphism exists that can map Figure 1a to Figure 2. Because Figure 1a (\( f_1(x) = x \)) is a line, to map it to any
function \( f(x) \), the transform function \( g(x) \) will be just \( f(x) \). For instance, to transform the line pictures in Figure 1a (\( f_1(x) = x \)) to the cubic pictured in Figure 2 (\( f_2(x) = \frac{2}{7} x^3 - 7x^2 + \frac{11}{7} x \)), the transform function must be \( g(x) = \frac{2}{7} x^3 - 7x^2 + \frac{11}{7} x \). This \( g(x) \) is not one-to-one and therefore has no inverse. In general, because our postpositivist model assumes a sign change in the derivative of the function, the postpositivist model would not be invertible and therefore a diffeomorphic mapping from Figure 1a to any postpositivist model based on our gleaned assumptions would not exist. We are forced to conclude that the mathematical models developed here for positivism and postpositivism are not qualitatively equivalent.

**DISCUSSION**

Today science educators are committed to helping students move away from an absolutist and rigidly cumulative view of the nature of science consistent with the early positivist movement toward an understanding of the tentative and revisionary character of scientific knowledge represented by postpositivist philosophers of science (American Association for the Advancement of Science, 1993; National Research Council, 1996; National Science Teacher's Association, 1998). Implicit in this concern is an appreciation and understanding of basic mathematical concepts such as linearity, nonlinearity, and rate of change. Although the field of mathematics can provide a number of graphic models that could be useful in describing the nature of scientific thinking and change over time, such models have not been developed for use in the classroom. In this paper, we provide one example of how formal mathematical models can be used to conceptualize specific epistemological commitments in science. The models developed in this paper could easily be transformed into activities that might be useful in helping mathematics and science educators develop a more sophisticated, formal, and explicit view or schema of the nature of scientific knowledge and scientific change.

Briefly, an instructional activity or unit for science educators (pre-service or experienced teachers) or undergraduate students might begin with a class discussion focused on the ways mathematicians deal with the rate of change over time (i.e., the concept of the derivative) in describing observations of the natural world. During these discussions, the different mathematical models described in this paper (e.g., linear, quadratic, cubic, logarithmic, and exponential functions) could be introduced, compared, and contrasted. During the next class period or periods, students (i.e., the science educators or undergraduates) are introduced to and discuss the philosophical positions referred to as positivism and postpositivism, perhaps through a series of vignettes. At some point, the students are asked to read selected excerpts from positivist and postpositivist scholars as homework and to characterize the development of scientific knowledge suggested by each philosophical orientation implied in the excerpts. These homework excerpts or readings should be chosen carefully and highlight how a positivist or postpositivist would characterize the concept of change as it relates to scientific knowledge or theoretical commitments over time. The homework assignment and follow-up activities could be guided by the following questions:

- Are the views of positivists and postpositivists similar or different as they relate to how knowledge develops over time in science? Explain your reasoning.
- Can the views of theoretical progress and development in science advanced by positivists be characterized using any or all of the mathematical models discussed in class? Explain which model(s) would or would not, in your opinion, be consistent with positivism.
- Can the views of theoretical progress and development in science advanced by postpositivists be characterized using any or all of the mathematical models discussed in class? Explain which model(s) would or would not, in your opinion, be consistent with postpositivism.
- Can the views of theoretical progress and development in science as described by positivists and postpositivists be distinguished using the mathematical models discussed in class? Explain your reasoning.
- Is there any value in attempting to characterize the views of theory development and progress in science advanced by positivists and postpositivists using the formal mathematical models discussed in class? Explain your reasoning.
- Are there any advantages and/or disadvantages in translating terms and concepts into their mathematical model forms? Can you think of other mathematical equivalents that could be used for these terms? Explain your reasoning. (This question and discussion should focus on the virtues of lexical precision at the expense of the loss of excess meaning.)
• Could you use these ideas in some form to teach your students about the nature of theoretical progress, change, and development in science or about the basic views and commitments of positivists and postpositivists? Explain.

This type of activity and its associated concepts could be made more or less sophisticated based on the type of students in the class. For example, students with little background and interest in mathematics may be better served by an activity that emphasizes the general relationship between positivism and an accretive view of theoretical development in science (shown in Figure 1a) and postpositivism and a nonaccretive view of scientific development (shown in Figure 2). Conversely, students with a strong mathematics background may be interested in discussing qualitative mathematics and the idea of mathematical equivalence and trying to demonstrate how positivism and postpositivism are either equivalent or non-equivalent using the more technical mathematical concepts elucidated in this paper. It should be emphasized that this brief discussion is not intended to represent a complete or comprehensive instructional activity, but only suggests a crude example of the instructional possibilities that can be linked to the ideas raised in this paper. Many of these ideas and concepts may be selected and isolated based on the abilities of the students and the aims of the instructor, course or program of study. This type of instruction, it should be emphasized, combines important mathematical concepts and principles with the central features of nature of science studies (theoretical stability, change, and progress) and therefore may represent content that could be taught by mathematics and science educators competent in philosophy. Further, this type of mathematical modeling is not restricted to science per se, but may be used in different content areas such as history (including the history of mathematics), psychology, literature, and economics, among other disciplines.

Some may readily object to using this level of mathematical formalism to describe the nature of scientific change and progress over time on the grounds that these ideas are fairly straightforward enough to discuss in plain English, thereby eliminating the need for abstract models. However, many adolescent (as well as adult) learners have difficulty with multi-modal thinking or the ability to consider and conceptualize antagonistic or opposing perspectives (Perry, 1970)—such as positivism and postpositivism. Further, epistemological and philosophical concepts often pose conceptual difficulties for students at all levels, making the development of explicit and formal models in these areas quite desirable. Indeed, the instructional virtue of formal models is suggested by Thom (1975), the progenitor of catastrophe theory, when he states that “as soon as a formal model is intelligible, it admits a semantic realization, that is, the mind can attach a meaning to each of the symbols of the system…” (p. 20, emphasis in original). Similarly, Polya (1981) notes: “Graphs, diagrams, or geometric figures used as symbols form a sort of mathematical language. To draw a figure, to express the problem in the language of geometric figures is often helpful” (p. 85). Latour (1988) suggests that the use of formal graphic models of representation (what he calls inscriptions) represent important cognitive strategies used by scientists to reduce their world of perception and experience into condensed, comprehensible and visual products that mobilize their ideas and theories. Latour's inscriptions— which include graphic models—provide a structured and organized “meaning-making” system with the aim of stimulating further analysis, reflection and refinement of ideas (metacognition) within the conceptual boundaries of a specific model (or inscription). In an instructional context, formal models have the potential to serve as compensatory instruction and cognition devices for students by allowing them to operationalize and examine their views and arguments in graphic form, which may encourage a higher level of reflection, abstraction, and meaning-making. Although mathematical modeling is common practice in the sciences, the use of mathematical models to describe the nature of scientific knowledge is a largely unexploited area—an area that may yield practical instructional fruit, as this paper has suggested.

ACKNOWLEDGEMENT

We wish to thank the participants of the Ninth Annual Colloquium on Research in Mathematics and Science Education (Spring, 2004) for their thoughtful insight and criticism, which led to many of the refinements and additions in this revised paper. Mathematics needs both individuals (to create new mathematics) and society (to accept, edit, and communicate mathematics) to develop. The meaning of mathematics is not found in the written symbols and formulas of mathematics, it is found in the shared understanding of human beings (Davis & Hersh, 1981/1998). As Hersh (1997) argues, “…mathematics flowers as a part of human culture” (p. 218).
References


ABSTRACT

The purpose of this pilot study, done as part of a doctoral independent study during Summer 2004, was to develop a deeper understanding of the theory of naturalistic inquiry and the process of devising research questions and defining appropriate methods to study those questions. The overall design and analysis procedures advocated in Lincoln and Guba’s Naturalistic Inquiry (1985) served as a guide for developing this pilot study and for analyzing and reporting the results. The specific focus of the pilot study was to gauge urban elementary teachers’ perceptions of the implementation process of TERC’s Investigations in Number, Data, and Space in order to begin to formally describe teacher responses using naturalistic inquiry. The purposive sample of three elementary teachers of grades one, two, and four was drawn from a mid-sized, culturally and linguistically diverse city school district in Massachusetts just finishing its first year of implementation. Each teacher was interviewed to gather data for analysis. Teachers were given the questions in advance, and interviews were audio taped and transcribed for review. Teacher responses were summarized and then analyzed using related literature, namely Shulman’s (1986) concepts of subject matter content knowledge and pedagogical content knowledge. Responses indicated difficulties with both mathematical content knowledge and pedagogical content knowledge during the implementation of Investigations despite participation in five to seven days of curriculum unit workshops accompanied by other forms of professional development. This pilot study concluded that, based on the needs of the three participating teachers, the district should review its professional development plans for year two of the implementation of Investigations, including incorporating more instruction of mathematics and pedagogy with access to support for teachers throughout the school year.

INTRODUCTION

National curriculum groups, such as the National Council of Teachers of Mathematics (NCTM), have called for reform in mathematics education (NCTM, 1989, 1991, 2000). The main reform idea is to promote mathematics students’ conceptual understanding rather than impart algorithms and procedures. To effect this change of mathematics instruction, the role of the teacher must be carefully reviewed and probably changed. An effective teacher is a facilitator of student discourse, referee of different methods and ideas, modeler of alternative ways, designer of open-ended activities and questions, and evaluator of appropriate materials for classroom use. There does not seem to be any issues with revolutionizing mathematics instruction; the questions seem to lie with how to bring the change about in an effective way (Sherin, 2002).

The NCTM’s Curriculum and Evaluation Standards for School Mathematics (1991) brought changes in published curricula. The National Science Foundation funded several groups to design standards-based materials for use in elementary and middle school mathematics classrooms. Standards-based mathematics curriculum materials were produced in response to the standards, and therefore differ from traditional textbooks series that merely aligned previously produced materials to the standards. Trafton, Reys, and Wasman (2001) define standards-based programs as comprehensive and coherent mathematics curricula that seek to develop ideas in depth, promote sense making, engage students, and motivate learning. As a result of ten years of piloting and editing these programs, many of these standards-based curricula are being implemented in districts across the country that are trying to increase student achievement scores on tests that are based on the mathematics standards.

There are challenges for the districts that attempt to implement standards-based mathematics curricula after decades of teaching with traditional textbook series. The mathematical content knowledge, the pedagogical content knowledge, and the teaching methodology required of teachers using standards-based curricula are different from those using traditional curriculum. Professional development seems a necessary component to the successful implementation of a standards-based program, but there are many questions regarding the content and delivery of the professional development.

REVIEW OF RELATED LITERATURE

What types of knowledge do teachers need to be successful mathematics teachers? Prior to Shulman (1986), it was generally assumed that teachers had the necessary content knowledge to teach their respective disciplines. Shulman suggested dividing content
knowledge into two categories: subject matter knowledge and pedagogical content knowledge. Subject matter knowledge includes the amount and the organization of knowledge, such as key concepts and facts, principles, paradigms, heuristics, and rules of proof. Pedagogical content knowledge is the understanding of what makes the subject matter knowledge understandable to others, including what makes it simple or difficult to learn.

Aubrey (1996) built upon Shulman’s work by finding that the range and depth of teachers’ subject matter knowledge directly affected their pedagogical content knowledge. Aubrey found that teachers who were deficient in subject matter knowledge did not have the capacity to expand upon student ideas or ask probing, open-ended questions spontaneously. Teachers’ subject matter knowledge was expressed in a variety of ways, including the design of the lessons, the actual instruction, the diverse representations used, and the links to other mathematical strands. Aubrey also stated that pedagogical content knowledge is shaped by teachers’ beliefs about mathematics. Therefore, strong subject matter knowledge and pedagogical content knowledge are essential to effective mathematics instruction.

With the understanding that subject matter knowledge has a role in effective mathematics instruction, researchers began to define factors influencing subject matter knowledge. Sanders and Morris (2000) tested British pre-service teachers using the Moriarty instrument and identified deficient areas of subject matter knowledge throughout the mathematics curriculum. The Moriarty instrument also provided data regarding confidence about mathematical content strands. Sanders and Morris found that students reported low levels of confidence for unknown or unrecognizable strands. Students who also erred on numeration, money, and computational examples cited low confidence as well. Pre-services teachers who failed the test were categorized by their responses to the test. One group was those who did not believe that content knowledge was a problem and tried to avoid mathematics. Another group was those who saw the lack of mathematical content as an issue, but did nothing about it. The third group consisted of those who recognized their lack of content knowledge and were motivated to improve. Therefore, the majority of the pre-service teachers who failed the math content test did not do anything to improve their subject matter knowledge. Sanders and Morris concluded that gaps in mathematical subject matter knowledge must be addressed; however, it is necessary to take into consideration the response pre-service teachers may have to discovering mathematics content deficiencies.

Like those in Sanders and Morris’ study who found their lack of subject matter knowledge debilitating, Bibby (2002) holds that many elementary teachers feel shame about their understanding of mathematics. She states that although the mathematical subject matter knowledge for elementary teachers is seemingly unproblematic, it is the traditional, absolutist method of mathematics instruction that causes shame in elementary teachers. Answers are either right or wrong, and the shame of being incorrect can cause lack of confidence or intense dislike of the subject matter. Teachers who hold intense emotional reactions to mathematics can become productive mathematics teachers by using their shame as a motivating factor to develop a holistic, connected view of mathematics.

Lowery (2002) suggested that improving mathematical subject matter knowledge of pre-service teachers may improve their confidence in mathematics and science. She worked with pre-service teachers to prepare them for elementary mathematics and science instruction. She found that when methods courses were rich in content and involved pre-practicum experiences, the pre-service teachers had significant gains in positive content and confidence to teach math and science at the elementary level.

Another strand of researchers focused on the effects of mathematical subject matter knowledge of pre-service and in-service teachers on attempts to reform mathematics instruction. Manouchehri and Goodman (1998) investigated whether the implementation of a new curriculum using a conceptual approach would help in the transition from an algorithmic-based series. They used observations and interviews and found that a reform effort showed some results when teachers felt supported emotionally and intellectually and had opportunities to develop both content and pedagogical knowledge. The researchers also found that the teachers’ lack of subject matter knowledge did affect their instruction even though the teachers were trying to correctly apply their pedagogical knowledge. For example, the teachers encouraged students to explain their thinking and to build on their conceptual understanding; however, the teachers did not always have the ability to respond to student discourse appropriately and guide students to expand ideas and generalize concepts.

Goulding, Rowland, and Barber (2002) identified deficiencies in British pre-service teachers’ subject matter knowledge and found a relationship between lack of apprehension toward content knowledge and insufficient planning and instruction. They noted that most
elementary school pre-service teachers have not specialized in mathematics and have limited mathematics coursework. The teachers have not been trained in making connections between mathematics content and pedagogy. The researchers concluded that teachers who lack subject matter knowledge do not take the necessary risks in discourse with the students and may not know how to answer student queries. They found that in order to be an effective elementary math teacher, it is necessary to have a systematic presentation of new ideas and to make explicit connections between different representations (visual, numerical, verbal) for students. After testing and observing pre-service teachers, they found a relationship between subject matter knowledge and competence in teaching numeracy. Those with low test scores were more likely to be rated incompetent numeracy teachers while those with high test scores were more likely to be rated competent numeracy teachers. The researchers also concluded that subject matter knowledge was not the sole defining factor of competency in mathematics instruction.

Investigating the relationship between subject matter knowledge and the ability to evaluate student work, Van Dooren, Verschaffel, and Onghena (2002) found a connection between teachers' subject matter knowledge and the way they evaluate students' answers and responses. They suggested that secondary teachers frequently awarded algebraic solutions more points than other correct solutions, placing more value on traditional solutions, while elementary pre-service teachers were more flexible in their acceptance of solutions and awarded points without favoring algorithmic solutions. However, elementary teachers also gave low points to solutions that they did not understand, despite the accuracy of the methods. This interplay of subject matter knowledge and pedagogical content knowledge is important to developing pre-service teachers because they will rely on what they know in order to teach and evaluate students.

Even if teachers are given a new curriculum that is designed to meet the requirements of the reform efforts, there is no guarantee that there will be a change in mathematics instruction, especially at the elementary level. Sherin (2002) claims that in a reform climate, teachers must learn while in the process of teaching. Teachers with limited subject matter knowledge do not have the understanding of the discipline of mathematics in order to change mathematics instruction. They do not necessarily have to “unlearn” mathematics; they just need to learn more content, namely syntactic knowledge of mathematics. Sherin holds that just because teachers receive new instructional materials does not mean that teachers use them in effective ways. Teachers do one of three things in novel teaching experiences, according to Sherin. They may transform the lesson by using their subject matter knowledge and pedagogical content knowledge to use the lesson, but use the materials in a different way than advocated by the curriculum designers. They may adapt, meaning that they learn the necessary content and do the lesson as written. Finally, they may negotiate, meaning that they learn new content but also change the lesson in progress in the classroom.

In order to successfully implement standards-based mathematics programs, Phillips, Lappan, and Grant (n.d.), as contributors to the Show-Me-Center, advocate for professional development that focuses on mathematical content and pedagogy that will allow teachers who are implementing standards-based curricula to adequately plan and teach. They hold that teachers need to increase their mathematical knowledge and in many cases, need to restructure their mathematical understanding from the rote knowledge associated with traditionally taught mathematics to an understanding of mathematics that identifies the interconnectedness of mathematical ideas and how concepts in mathematics build upon each other. Teachers need more training than the mere methodology of the new standards-based programs; they need fluency with mathematics content and pedagogical content.

The ARC Center (2000), a collaboration between the Consortium for Mathematics and Its Applications and the three standards-based programs (Investigations, Everyday Math, and Trailblazers), has published case studies that describe the implementation of Investigations in New York City, Madison, AZ, Marshfield, MA, and Hudson, OH. Although these districts represent vary greatly (suburban, urban, affluent, economically challenged, monolingual, and multilingual), some common themes can be gleaned from the ARC Center’s work. The case studies demonstrate that the teachers in these districts needed significant support during the implementation, especially in the first year. The types of support that were reported as helpful to teachers include mathematics study groups, math lead teachers/mentors/facilitators, workshops, and graduate coursework. The ARC Center also found that these districts began the implementation process with a pilot year with a small group of volunteer teachers. In the next years of implementation, these teachers served as the mentors for the new teachers joining the implementation process. This volunteer piece of implementation also caused challenges for some of the districts, due to the inconsistency of the math programs for students.
across the district. Districts that mandated Investigations use found that the consistency was strong, but that it was difficult to ensure that all teachers were effectively using the program and communicating the goals and philosophy of the Investigations to the parents. In the implementation case studies, all districts expressed the challenges relating to changing views of mathematics. Principals needed to develop new understandings of what they would see in a standards-based mathematics classroom. Teachers needed to develop new understandings of mathematics content and methodology in order to carry out Investigations sessions. Districts reported the financial cost of successfully implementing Investigations as one of their main challenges. The cost of the program’s materials, the professional development providers, and the teachers’ stipends were overwhelming, and in some districts, were cut, leading to more difficulties in the implementation.

RESEARCH FOCUS/QUESTIONS

The literature demonstrates that developing effective mathematics teachers is a complex endeavor. Teachers of mathematics must show confidence and flexibility in their understanding of mathematics and must have a positive attitude toward the discipline. Additionally, teachers implementing a standards-based mathematics program must have the necessary mathematical content knowledge, pedagogical content knowledge, and the support needed to implement a new methodology. As evidenced from the ARC Center’s implementation case studies, this is a challenging task.

The purpose of this pilot study was to develop a formative evaluation, which as defined by Lincoln and Guba (1985), is aimed to inform improvements or refinements to what is being evaluated (p. 227), which in this particular case is the implementation of the Investigations program in a district unlike the four studied by the ARC Center.

The following questions describe the focus of this pilot study and attempt to define the necessary boundaries and parameters for inclusion and exclusion as outlined by Lincoln and Guba (1985, p. 226-7). What are teachers’ perceptions of the implementation of the standards-based Investigations in a district unlike the four studied by ARC Center? What are teachers’ perceptions of the accompanying professional development? What are teachers’ perceptions of the implementation and professional development impact on their teaching?

Because these questions originated from a complex problem situation, it is imperative to design a naturalistic inquiry as advocated by Lincoln and Guba. In this particular problem situation, multiple factors include the standards-based Investigations elementary mathematics program, the accompanying professional development, and the teachers’ perceptions of both the implementation and the program.

RESEARCH METHODOLOGY

Because the pilot study focused on teacher perceptions of the implementation process, it was necessary to devise an appropriate plan to gather data. Lincoln and Guba (1985) advocate purposefully choosing participants so that the researcher can identify “in whatever way one can, a few members of the phenomenal group one wishes to study” so that the selection “will provide the broadest range of information possible” (p. 233). Therefore, three teachers were purposefully chosen to participate in a fifteen to twenty minute interview with me. To gather a larger range of data, I chose teachers from different grade levels. These first, second, and fourth grade teachers were from two different schools in a northeastern, urban district in Massachusetts. Teachers were asked to review a consent form letter and sign to demonstrate agreement to participate. In order for the teachers to become familiar with the interview issues, they were given a list of six questions one day prior to the interview (see Figure 1).

All participants were encouraged to write notes on the interview questions sheet for their use throughout the interview, and all participants reported to the interview with copious notes. All interviews were conducted in a one-on-one setting and were openly tape-recorded. Interviews were audiotaped to provide fidelity of the data recording mode (Lincoln & Guba, p. 240), and the participating teachers were assured of their privacy to avoid a threat to candidness as

Figure1. The Interview Questions

<table>
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<th>Interview Questions</th>
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<tr>
<td>1. What is your general teaching background? What is your educational background?</td>
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<tr>
<td>2. Briefly describe your teaching.</td>
</tr>
<tr>
<td>3. What are your likes and dislikes of the Investigations program?</td>
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<tr>
<td>4. What were your expectations of the professional development that accompanied the implementation of Investigations?</td>
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<tr>
<td>5. Did the professional development meet your expectations? Explain how it impacted your teaching and the implementation of the program.</td>
</tr>
<tr>
<td>6. Please share any other information you wish about the implementation of Investigations.</td>
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described by Lincoln and Guba (p. 241). Transcripts of
the recorded interviews can be found in Appendix A.

DESCRIPTION OF PILOT STUDY DISTRICT
AND SAMPLE

This pilot study took place with classroom teachers
from elementary schools in a mid-sized, culturally and
linguistically diverse city in Massachusetts. There are
fifteen elementary schools and nine middle schools in
the district. The researcher selected as cases for study
from two of these schools, three classroom teachers of
grades one, two, and four who each volunteered to be
interviewed. The pilot study took place over the sum-
mer of 2004. Two of the three participating teachers
were teaching in the district’s elementary summer
school program, and their interviews took place in their
summer school classrooms. The third teacher did not
teach during the summer, and her interview took place
in her classroom from her regular school year.

The district’s student population is culturally and
linguistically diverse. Approximately half of the student
population is Caucasian while roughly one-quarter is of
Asian and one-quarter is of Hispanic descent. English is
not the primary language spoken in the homes of over
half the students in the district. The district has adopted
an English immersion program to afford English lan-
guage learners support in the mainstream classrooms;
therefore, roughly one-quarter of classroom teachers in
the district are bilingual or have completed coursework
specializing in teaching second language learners. The
classrooms have been heterogeneously grouped at the
elementary level, so that each classroom has been com-
pose of members of different ability levels, language
backgrounds, and socio-economic status.

In addition to adopting an English immersion pro-
gram, the district has a voluntary desegregation policy.
Each school’s population is reflective of the racial make-
up of the city. Parents choose the schools that they
would like their children to attend. They are given their
first choice unless those schools have waiting lists.

This study focused on the implementation of the
Investigations program in grades one, two, and four. All
recommended materials, such as the teacher’s manuals,
resource books, student activity books, calculators,
mathematics manipulatives, and assessment guides,
were purchased and disseminated to each classroom
teacher in the district.

HISTORY OF DISTRICT’S MATHEMATICS
INITIATIVE: WHY IMPLEMENT A NEW PROGRAM?

Like many other urban districts in Massachusetts,
the district demonstrates lower Massachusetts
Comprehensive Assessment System (MCAS) scores
than its suburban neighbors (Massachusetts
Department of Education, 2004). The fourth grade
mathematics scores on the MCAS are below the state’s
average. Eight of the elementary schools have been
labeled “underperforming” by the Massachusetts
Department of Education and are following school
improvement plans to increase student achievement.

After completing a major overhaul of the district’s
language arts curriculum and instruction, the district
began a new initiative—adopting new mathematics
curricula for the elementary and middle schools. In the
early 1990s the district had purchased new mathemat-
ics series for the elementary level. Each school reviewed
different traditional textbook series (SRA, Scott
Foresman/Addison Wesley, and Silver, Burdett, and
Ginn) and chose one of the three to purchase and use.
The teacher editions, student textbooks, consumable
workbooks, and manipulative kits were issued to each
classroom in the district; however, use of the textbook
series was not mandated by the district’s curriculum
office. Therefore, teachers could use a variety of materi-
als in order to meet the objectives and standards of the
district and the standards outlined by the
Massachusetts Mathematics Curriculum Frameworks
(2000).

With the advent of the MCAS, Lowell had evidence
that its fourth grade students, tested at the end of their
elementary school experience, were not meeting the
evaluation standards set forth by the Massachusetts
Department of Education. In fact, the majority of the
district’s fourth grade students continually scored in the
“Needs Improvement” or “Warning/Failure” levels
(Massachusetts Department of Education, 2004). Figure 2 (next page). provides the percentages in each
performance level for four years: 1998, 1999, 2000,
and 2001 as posted by the Massachusetts Department
of Education website. Roughly forty percent of students
failed the Grade 4 Mathematics MCAS tests each year,
and roughly forty percent more tested at the need
improvement level.

In response to these low, stagnant test scores, the
district, under the direction of the deputy superinten-
dent, hired a district-wide K-12 mathematics coordinator
to work on developing a plan for increasing student
achievement. In the 2002-2003 school year the mathe-
ematics coordinator led the mathematics curriculum
committee, comprised of elementary, middle, and high school teachers and administrators as well as the district-wide professional development coordinator. Additionally, the district changed the role of mathematics lead teacher from teachers who worked with a local university professor who paid stipends from a university grant to stipend positions of four extra hours each week funded by the district. The major task of the mathematics curriculum committee and the mathematics lead teachers was to review mathematics programs for the next school year in grades kindergarten through grade 8. The groups reviewed the sample materials of a variety of programs, including traditional textbooks series and standards-based programs.

Traditional textbooks were quickly eliminated from contention. Namely, committee members were concerned about replacing one textbook with another to lead to the same troubling test scores. Of the elementary standards-based programs, namely Trailblazers, Everyday Math, and TERC’s Investigations, only Everyday Math and Investigations were selected for the final evaluation. The district developed a scoring rubric for teachers and administrators to use to closely evaluate the programs. The rubric assessed such items as richness of curriculum; teacher-friendliness; and organization. Mathematics curriculum committee members and math lead teachers were required to score both programs during a series of meetings at the central office. Other teachers and administrators were invited and encouraged to review and evaluate the program materials that continued to be stored at the central office.

At the end of the evaluation period, the mathematics coordinator and the professional development coordinator decided to purchase Investigations for every kindergarten through grade 5 in the district. Evaluators had scored both programs similarly, but there were two factors that were ultimately determinative. The middle school group had chosen Connected Mathematics, which is a grade 6 to grade 8 standards-based curriculum. Investigations, as a kindergarten through grade 5 program, created an easier sequence than Everyday Math, which is a kindergarten through grade 6 curriculum. Second, Everyday Math was developed as a spiral curriculum in a chapter format while Investigations was developed as a unit-based curriculum. The mathematics coordinator and the professional development coordinator thought that it would be easier for teachers to implement Investigations because they could use the curriculum’s units as replacement units, as opposed to implementing all of Everyday Math in the first year. Additionally, the professional development coordinator had assisted with the implementation of Investigations in the district in which she had previously worked and had positive experiences with the professional development consultants associated with Investigations.

All Investigations curriculum materials were purchased for each classroom teacher from kindergarten through grade 5 throughout the district in April, and materials were delivered to the schools before the end of the school year.
of the year for use during the next school year. The professional development coordinator also contracted for consulting professional development services, and the mathematics coordinator worked on in-district mathematics support.

The district instituted a variety of professional development opportunities for teachers in the district. A mathematics lead teacher model, in its third year, has continued to support the implementation of the programs. Each school in the district has one math lead teacher who is either a classroom teacher or a Title I classroom support teacher. Mathematics lead teachers receive a stipend in exchange for added responsibilities to promote the mathematics initiative. They handle logistical issues as well as professional development issues. Their responsibilities include coordinating professional development schedules, modeling lessons for teachers in the building, disseminating information from the district to teachers and administrators, coordinating and leading parent information sessions and family mathematics nights, planning and leading voluntary mathematics study groups for teachers, and mentoring classroom teachers in the implementation of the programs. Mathematics lead teachers attended six daylong workshops during the school year in addition to monthly meetings with the district’s mathematics coordinator.

The workshops were conducted by a team of university consultants who are considered experts in the implementation of the Investigations program. Teachers of kindergarten through grade two attended a three-day workshop during the summer, totaling roughly 20 contact hours. One set of teachers attended the workshop series in June; others opted to attend in August. Teachers received per-diem pay for these three days. Participants were separated by grade level taught. Consultants explained the philosophy and the goals of Investigations, and then introduced the teacher’s unit manuals and the assessment guide. The remainder of the workshop days were devoted to introducing the first unit that the teachers were scheduled to teach. Teachers of grades 3 through 5 attended a one-day workshop in August to become acquainted with the routine component of Investigations, called Ten-Minute Math. Again, teachers were paid per-diem for their attendance.

Teachers of kindergarten through grade two were required to teach four units, one for each quarter of the school year, from the Investigations program during the 2003-2004 school year. The university consultants trained the teachers prior to the teaching of the units.

Teachers attended a daylong training in October for the second quarter’s unit, in November for the third quarter’s unit, and in December for the last quarter’s unit. Teachers of grades 3 through 5 were required to teach two units, one for the third quarter and one for the last quarter. These teachers continued to use the former textbook series for the first semester of the school year. These teachers received training in December for the third quarter’s unit and in February for the fourth quarter’s unit. Substitutes covered the teachers’ classroom responsibilities for the professional development days. Teachers at each grade attended the same training with the same trainer, but were placed in two groups over the course of two days to afford small training groups. Teachers were required to bring the teacher’s manual for the unit and the program’s assessment guide to the training. The training focused on introducing the activities and the games of the units to the teachers in addition to explaining the general goals and objectives of each unit.

**CASE STUDY: TEACHER A (GRADE 1 TEACHER)**

Teacher A, a first grade teacher, is an eighth-year teacher, who other than her year-long student internship experience, has taught solely in the district. She teaches in a single-strand elementary school and serves as the school’s only first grade teacher. The school has an English immersion program, and Teacher A teaches English language learners whose native languages include Spanish and Khmer. Teacher A has a bachelor’s degree in English and a master’s degree in special education. She has recently enrolled in a doctoral program, concentrating in literacy. Teacher A serves as a mentor to new teachers in the building and has been a cooperating teacher to many student teachers from the local university.

Teacher A holds developmental theory as an important influence on her teaching. She believes that children learn by doing and that the activities and lessons must be meaningful to children and authentic in nature. Teacher A stated that assessment must also be authentic and that children should be given various opportunities and forums to demonstrate their learning.

In terms of the Investigations implementation, Teacher A cited many likes and dislikes. The overall philosophy of the Investigations program aligns with her thinking about meaningful, active learning. She feels that the Investigations program is inquiry-based and allows students chances to investigate mathematical ideas. However, Teacher A is concerned about the lack of direct teaching advocated by Investigations. In her
experience this year, she found that students did not always make the connections expected by the program and wondered if it was even developmentally appropriate to expect first graders to make these discoveries. Additionally, Teacher A had to supplement the *Investigations* program to cover what she considered to be important skills for first graders, such as time and money. Lastly, Teacher A explained that the pace of the *Investigations* program was quite slow and that she barely finished the four units mandated by the district. She cited concerns about fitting in the six units required for next year, especially since she worries that she will also need time to supplement with other materials to cover all of the first grade standards and benchmarks.

Teacher A spoke positively of the accompanying professional development offered by the district. She felt that the three-day summer workshop helped her form the foundations of *Investigations* and that with those basics, such as how to use the materials and manuals, she was able to navigate through the implementation year. Teacher A wished that the professional development offerings included more about the theories underlying the development of the program, but also thought that may not be an appropriate use of training time for all teachers in the district. Teacher A expressed concern that she was removed from her classroom for three days during the school year for unit training. She felt that once she had the basics, she could have figured out the sessions’ activities and expectations of those units on her own, without losing time with her students.

In all, Teacher A appreciates having *Investigations* as a resource to use for mathematics instruction, but is concerned about the mandate to use it exclusively. The professional development did impact her ability to access the materials quickly, but did not offer her the theoretical perspectives she sought.

**CASE STUDY: TEACHER B (GRADE 2 TEACHER)**

Teacher B, a second grade teacher, has been teaching for roughly ten years. She taught in another state for eight years and has just finished her second year of teaching in Massachusetts. Teacher B earned a bachelor’s degree in Child Study holds a valid teaching certificate. Teacher B taught second grade during the district’s implementation year but, as a permanent substitute, is unsure about her position next year. She has taken a few graduate courses in specific areas that she felt would improve her teaching and the learning of her diverse students. Teacher B describes her teaching style as one that assesses what students already know, what they need to know, and what the best method is to address their unique learning styles. She states that she uses a variety of groupings, such as small group, whole group, centers, and one-on-one experiences. Teacher B cites her appreciation of the think, pair, share concept and tries to afford her students the opportunity to share their strategies and ideas.

Regarding the *Investigations* program, Teacher B described many positive aspects. She explained that the structure of *Investigations* (the launch, explore, summary process) aligned with her belief in allowing students time to think about and share their understanding. Teacher B appreciated the open-ended nature of *Investigations* and that students could devise multiple methods of solving problems. She felt the activities were hands-on and noted that her students enjoyed using manipulatives. Teacher B also saw her students using concepts throughout the entire school year and building on their ideas, instead of learning concepts in isolation and never revisiting them.

Among the negative aspects that Teacher B described was the wordiness of the teacher manuals, which was something that she felt became easier to read over the school year. She described the program as requiring a lot of preparation time, which was an aspect that she did not see becoming easier over time. Teacher B noted concern about the lack of practice page support and explained that she needed to create *Investigation*-like practice pages for homework. The exclusion of more traditional strategies also troubled Teacher B, who feels that some students’ learning styles warrant the inclusion of these strategies.

In terms of the professional development that accompanied the *Investigations* implementation, Teacher B stated that the workshops offered by the university consultants were helpful, but the mentor hired by her school did not assist her with the challenges of implementation. Teacher B described the unit workshops as a time to learn about the manuals, the lessons, and the games, and it gave her the chance to think about what could be problematic when teaching it so that the trainer could help her troubleshoot. Teacher B expressed frustration with the timing of the training however; she stated that the workshops were held months before she was actually scheduled to teach the units. She felt it would have been more effective if she had the training right before the unit was taught.

Teacher B expressed great dissatisfaction with the mentor/consultant contracted by her school upon the recommendation of the professional development coordinator. Teacher B had hoped that the mentor would model lessons in the classroom or at least for the
teachers. She had also hoped that the mentor would be able to help her troubleshoot problems that arose in her classroom as she was implementing the sessions. One particularly unsettling issue for Teacher B was her students’ lack of understanding of place value. She was concerned that students could still solve the story problems and complete the activities, but that they didn’t make the necessary connections to fully understand the base system and its relationship to the operations. Teacher B was also concerned that her students continued to use just one strategy to solve problems, rather increasing efficiency and accuracy. Teacher B believed that a stronger mentor might have been able to help her with these difficulties, as would monthly meetings with her fellow second grade teachers.

CASE STUDY: TEACHER C (GRADE 4 TEACHER)

Teacher C, a fourth grade teacher, is a fourth-year teacher who has taught solely in the district. Teacher C taught preschool, kindergarten, and second grade in her first three years. This was her first year teaching fourth grade. She teaches in a single-strand elementary school and serves as the school’s only fourth grade teacher. The school has an English immersion program, and Teacher C teaches English language learners whose native languages include Spanish and Khmer. Teacher C has a bachelor’s degree in English and psychology and a master’s degree in elementary education. She holds certification in elementary education, early childhood education, and reading (provisional).

Teacher C describes her teaching as “constructivist,” namely that students interact with her, with materials, and with each other to build ideas that fit into their schemata. She utilizes small group instruction to meet this goal. Teacher C tries to serves as a facilitator of learning rather than provider of information.

In terms of the implementation of Investigations, Teacher C appreciated the program’s use of manipulatives throughout the hands-on lessons. She felt that “meshed” with her teaching style as she already felt that she designed activity-based learning opportunities for her students. She found, though, that students could become bored with the session because the program called for constant repetition of activities or games. Teacher C’s primary issue with Investigations was that the program called for students to be metacognitive, yet the students had difficulty expressing their thinking about their thinking. Teacher C states that at times she was unsure of the connections the students were supposed to make and that often, students knew the answers or the process, but they did not understand why the process and solution worked.

In terms of the accompanying professional development, Teacher C had low expectations for the workshops, so she felt they were better than what she had expected. She stated that the workshops allowed her to be in the place of her students and use a hands-on approach to access the material in the teacher’s manual of the units. Teacher C appreciated this time because later, as she taught the unit to her own class, she remembered how the trainer addressed student queries or concerns and was able to use that to help her students. Teacher C found the teacher's notes and dialogue boxes scattered throughout the teacher’s manual to be helpful in preparing for lessons. Additionally, Teacher C utilized the mentorship of the school’s math lead teacher who helped her plan lesson summaries and assess student work. However, Teacher C stated she would have appreciated the opportunity to meet with other fourth grade teachers who were also implementing the same units, something that was difficult to do in a single strand school.

ANALYSIS AND CONCLUSIONS

Though only three teachers were interviewed for the purposes of this pilot study, some themes emerge from their experiences that relate directly to the literature and thus, may impact the district’s next steps in the implementation of Investigations and the accompanying professional development. These themes include the need for continued development of the mathematical content knowledge in teachers, the lack of pedagogical content knowledge in some teachers despite training, the need for strong support for teachers during the implementation process, and the need to monitor teachers’ attitudes toward teaching and learning mathematics.

One theme that emerges is the need for continued development of the mathematical content knowledge in teachers so that the teachers have the flexibility and fluency required to be effective mathematics teachers. To highlight, one teacher expressed concern that she and her fourth grade students did not understand why some of the processes they tried worked. If the teacher has difficulty with the subject matter knowledge, it will dramatically impact the teacher’s pedagogical content knowledge (Shulman, 1986) and thus, her ability to effectively summarize Investigations lessons.

Another teacher interviewed for this pilot study described incidents of insufficient pedagogical content knowledge. Teacher B explained her fear that her stu-
students didn’t understand place value at the end of the year. Although she had done every session in the teacher’s manuals, some students still relied on a single method to solve problems and were unable to make connections between place value and the tasks required of them. Teacher B had hoped her mentor would assist her with improving instruction, yet that did not occur. The district may seek to improve its support of teachers, especially in how to use their mathematical content knowledge to anticipate student difficulties, plan appropriate lessons, and ask guiding questions so that the students truly have the opportunity to make these mathematical connections.

The lack of mathematical content knowledge and pedagogical content knowledge also leads to the absolutist view of mathematics. In the ARC Center’s implementation stories (2000), they found that this absolutist view of mathematics held by some teachers, administrators, and parents posed a challenge to the implementation process. For example, in this pilot study, a teacher expressed concern that some students learn the “old-fashioned, rote way” and that the inquiry-based methods did not work for these students. If teachers do not view mathematics as a dynamic, creative, interconnected discipline instead of a collection of procedures and rules, the district’s move to implement a standards-based program will have difficulties like those districts outlined by the ARC Center. The district may choose to provide teachers with the professional development necessary for them to develop an understanding of the nature of mathematics that is congruent to the standards-based movement so that these teachers do not continue teaching mathematics as they themselves learned it.

As Sherin (2002) theorized, just because teachers are given a new mathematics curriculum does not mean they use it as intended by the curriculum developers and the professional development providers. This seems to be the case with all three of the teachers interviewed for this pilot study. The Investigations authors and the university consultants advocate for the exclusive use of the program with absolutely no supplementation, yet all three teachers used supplementary materials throughout the school year. Secondly, the program developers state that the summary piece is the most crucial aspect of each session. It is during this time that the teachers give students opportunities to share and to listen to the strategies of their peers. Teachers are supposed to guide students with meaningful questions that will help students expand their thinking. Because the professional development offered focused mostly on the hands-on activities in the Investigations program, these three teachers focused mostly on the hands-on piece and not how to appropriately guide students through the explorations. Perhaps now that the teachers have received the unit training in the first year, the professional development opportunities for the second year can be expanded to included teacher questioning and assessment.

The three teachers who volunteered to participate in this study expressed the desire to promote student learning and interest in mathematics. However, after the first year of implementing Investigations, they also expressed a lack of confidence in teaching mathematics effectively using these materials. If the implementation is to be successful, more supports must be in place to assist these teachers and others like them.

References


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Transcripts of the Interview with Teacher A
Monday, July 26, 2004

R=Researcher
A=Teacher A

R: Thank you for participating, Teacher A. I guess we’ll start the interview off with what is your general teaching background?

A: Well, I have a master’s in special education which concentrated on traumatic brain injury and early childhood. I have been working in the same school system for about 8 years now. I started out working as a special education teacher for three years in an inclusionary setting and for the past five years, I’ve taught…let’s see…I’ve taught kindergarten, first grade, and second grade. For the majority of years I’ve taught first grade; actually every year I’ve taught first grade. Some of those were multi-age classrooms. I’ve also worked with preschool while I was getting my master’s degree in education. I’ve worked with severe special needs students; I’ve worked with ESL students. I’ve worked with students from various socio-economic backgrounds.

R: Thank you. Could you describe yourself as a teacher or say, your teaching style?

A: I am really committed to a developmental approach to education that values child-centered activities to facilitate learning. I feel strongly that children should be valued and respected in their classrooms. They should be given a lot of opportunities to show how or what they are learning, not just by paper and pencil tasks, and that the activities and curriculum should be meaningful and authentic.

R: Thank you. As you know the district you are in has implemented *Investigations* and you are in your first year with that process. The first question about this is: what are your likes and dislikes that you have after your first year of Investigations?

A: Some of the positives are that it is, to a large extent, inquiry-based and there’s a lot of opportunities for children to explore. There’s a lot of emphasis on using manipulatives in a meaningful way and this is something that I think is important, especially for young learners, since I teach first grade. Some of the negatives are that I think that the pace is rather slow. It spends a lot of time on one task. I also think that the programs expects children to discover and understanding about math concepts without direct teaching and that this is really something that is not developmentally-appropriate for some children at the younger ages. It also is missing some important skills. For example, there is really very little time spent on time and money, which are two big concepts for first graders.

R: In terms of the implementation of *Investigations*, the district conducted a series of professional development opportunities for teachers. Could you please describe your participation in the professional development, what you expected, and did it meet your expectations?

A: I participated in a three-day workshop in the summer and then two subsequent workshops on different days throughout the school year to help with the implementation of the different units that I was expected to use in the classroom this year. I found that my expectations were pretty congruent with what I was expecting. I expected that they would go over the teacher manuals and how to do the different unique things that go on in *Investigations* and they did a pretty good job with that. I guess one of the things that I found lacking was the, you know, there wasn’t a lot of emphasis on what theory and research drives that particular program. But, I also think that given the time constraints...
A: I guess that I see Investigations as being a better program if it weren’t an exclusive program. There are opportunities to supplement with other things, and that is something that I had to do in my classroom. But, I know that the directives from the central office were very clear about using Investigations exclusively. I know that some teachers were not allowed to use supplemental things and that does a real disservice to children. I am also concerned about the fact that we did not use all of the units this year. We used 4 out of the 6 and it took us all year to do those. We struggled to get them in because, as I mentioned before, the pace was really slow. They go over certain concepts repeatedly before they move on, to the point where children, if you follow the program to a “t” then the children begin to get bored with certain activities. I think that’s about it.

R: Thank you for your participation.
teaching all the time. Dislikes! Lot of prep that I didn’t anticipate because it’s the first year. There’s a lot of prep each year no matter what year you’re doing it. There’s not a lot of practice page support. I did a lot of my own, using the philosophy of *Investigations*. Sometimes it took a lot more time that they allotted for it for that day. In *Investigations*, some of the other kids who need a different style, say like a more old-fashioned, rote way, can’t learn it that way. So, sometimes it was really frustrating to try to teach the kids the *Investigations* way when they weren’t getting it. I was frustrated by the mentor we were given at our school. I felt like there wasn’t a lot of support. She was just learning it. She had never taught it before so it was disappointing to me as an educator teaching it for the first year without a good support. I would like more time for teachers to meet who are just learning it to prepare for that month before they do it.

R: Well, you’ve answered some of this already, but if you would like to add anything else… What did you expect of the professional development? So, you said that the workshops were good but the mentor piece was lacking…

B: The mentor piece was lacking. If it had been someone who had taught it and could troubleshoot it with us as opposed to us making up the agenda and have us to come up with ideas and us do everything. It was not helpful troubleshooting. It didn’t help me teach *Investigations* the way I could have, I feel. I expected how to plan the lessons, what was the philosophy, how does it organize this, how was I going to teach it effectively for kids who learn the old-fashioned way of learning? Was it going to give me an effective overview of the program so I would feel comfortable? It is going to tie in with the MCAS teaching? *(Looking at questions given to her the day before)* Number 5: Did it meet my expectations? Absolutely. Did a lot of lessons, did a lot of hands-on in those workshops, did some troubleshooting right then and there. The only disappointing part was we didn’t get to teach the unit right after. We were given the training months before we even taught it. So that piece kind of fell apart. It has impacted my teaching. It has allowed me to look at different curricula from different areas and makes me think about how I could teach it in a more open-ended way, with the share out piece, with the launch, explore, summary—the framework that *Investigations* uses. It’s helpful for other activities and other curriculum areas. It also did a good job of helping me think about objectives for each day. What was I going to teach that day and what would I expect to see at the end of the day from students? It was user-friendly that way and very structured.

R: Thanks. Was there anything else you wanted to share that I didn’t ask you about in terms of the implementation? Things that you think are important for perhaps policy-making for the district?

B: As a first year *Investigations* teacher, I would like to see more lessons demonstrated if possible. If that could be put into the budget that would be very helpful. They could come into the classroom or just do it for us, the classroom would be better. They should give us more effective mentors. One of my biggest challenges at the end was place value. I didn’t know how to teach it because the kids weren’t getting it with *Investigations*. They could solve problems using their own strategies, but they were able to explain or talk about place value. You have those kids how are still drawing pictures in June and they don’t move on because the philosophy says they can do it the way they learned. So, I am a little confused like that. I would love to have a workshop of selecting work for portfolios and how you do assessment data gathering. It was a small amount of time during the workshop schedule. I guess that’s all.

R: Thank you, Teacher B.

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**Transcripts of the Interview with Teacher C**

*Tuesday, July 27, 2004*

R=Researcher  
C=Teacher C

R: Thank you for participating, Teacher C. To begin, I would like to ask you about your general teaching background and your educational background.

C: I have been teaching in the early childhood and elementary school sector for four years. The population I work with is culturally diverse, primarily Caucasian, Hispanic, and Cambodian students from predominately lower income, non-traditional households. I have a bachelor of arts in English and psychology and a master’s in education in elementary education with certification in early childhood and elementary education and almost in reading, just pending additional classwork. I
taught kindergarten during my student teaching, which continued after that in the same school for the remainder of the year. Then I taught second grade in another school in the district. Then, I taught kindergarten and pre-school for a year in another school in the district and then returned to that school to teach a third and fourth grade combination.

R: Thank you. Could you please describe your teaching?

C: My teaching instruction is based on small group, hands-on, constructivist learning where students interact with each other and the material to be learned and I suppose a bit with me, too (said jokingly) So that students can begin to understand and integrate the new information into their schemas. I try to be a facilitator of the students' learning rather than an instructor.

R: Thank you. As you know the district you are in has implemented Investigations and you are in your first year with that process. The first question is what are your likes and dislikes that you have after your first year of Investigations?

C: Investigations incorporates the use of manipulatives and is most definitely hands-on, so the program fit well into my style of teaching. I did feel that some of the lessons were too drawn out, seemingly repeating the same exercise over and over and over. I noticed that the students seemed bored at different times. I found that students were not always able to transition from the concrete, cognitive to the metacognitive. They were able to solve a problem, but they couldn't or weren't able to form a reason or a theory or a formula—students got the what, but not always the why. To be honest, sometimes I had no clue either! It was very difficult for me to make the connections and that made it difficult for the students to make the connections too.

R: You received professional development with this process. What were your expectations of the professional development that accompanied the implementation of Investigations?

C: My expectations of the professional development for Investigations' implementation were a general introduction of the program, like an overview, and perhaps a breakdown of each particular unit--I unhappily expected mostly a lecture format.

R: Did the professional development meet your expectations? Did it impact your work as a fourth grade math teacher?

C: The professional development exceeded my expectations, which you already know, were quite low anyway. They set up the training as if the teachers were the students, so I could experience the hands-on approach myself. When the class worked on a unit I was familiar with, I remembered some of the questions I had and others in my group had during the workshops and I was able to anticipate any problems the kids may have. Making the games to be played and then actually playing with them made the program much more understandable and accessible rather than just watching a slide show or reading a few chapters.

R: Did you have any other comments you wanted to add about the Investigations implementation that I didn't ask specifically?

C: Being in a single strand school, I found myself questioning aspects of the units at times during implementation of the program. Was I spending too much time on making arrays, how could I assess whether the students were tying in the arrays to multiplication facts, you know. Fortunately, having a knowledgeable and generous math lead teacher with considerable expertise helped, but I would have also liked and, you know, benefited from being able to speak more frequently with another fourth grade teacher going through the same lessons. The Investigations book presented scenarios with possible student questions and problems, and those provided insight as well. Also, I appreciated having all materials needed for a unit available in the kit; saved lots of time and money.
ABSTRACT
This paper is an outcome of an independent study course I took on Qualitative Research during the summer of 2004. The primary reading material for this course was *Naturalistic Inquiry* by Yvonna S. Lincoln and Egon G. Guba.

The focus of my research was to investigate the attitudes of college students toward homework in mathematics courses. As a college mathematics professor I am very interested in understanding student motivation toward homework and also their study habits concerning mathematics. In carrying out this study I held separate interviews with four college students taking an Algebra and Trigonometry summer course at a state college. During these in-depth interviews we discussed all aspects of both written and online mathematics homework.

In carrying out this research I learned how to conduct a qualitative study using the methods of "Naturalistic Inquiry" and the importance of using a human as the instrument of data gathering. The results of the research show that most students want homework to be collected and graded, do not spend enough time on homework, and liked the idea of online homework.

There is a general concern among college faculty that many students do not adequately complete homework assignments. This is especially critical in college mathematics where the pace is accelerated and students are expected to complete a significant amount of homework. In many ways learning mathematics is like learning to play a musical instrument. The teacher can show the student the steps, or where to place the fingers, but the material and the skill can only be learned through practice. As a college professor teaching mathematics with a specialty in remedial mathematics, I am interested in understanding student motivation toward homework and also their study habits concerning mathematics. This qualitative investigation was carried out to provide some insight in these areas.

In order to effectively conduct a qualitative research study I enrolled in an independent study during the summer of 2004 on Qualitative Research. I met with the Professor only once during the duration of the independent study and during this time I read a book by Yvonna S. Lincoln and Egon G. Guba entitled *Naturalistic Inquiry*. The interview method of gathering data and the use of the case study reporting format described in great detail in this book are the basis of the work reported herein.

LITERATURE ANALYSIS
It has been reported that the college freshmen class of 2003 spent less time studying and more non-academic time on computers in their senior year of high school than any previous entering class (Marklein, 2003). A few causes for the decline in study time are that more students are active in extracurricular activities, more students are working, and many students do not know how to study effectively. Some research places the blame for poor study habits both on the teachers and the students. According to Young (2002) researchers found that professors may be partly to blame because they have lowered their standards for acceptable work. The resulting grade inflation means that students do not have to work as hard for their grades as in the past (Marklein, 2003). Students that succeed in high school with very little work feel that they should receive good grades in college when with the same effort. Young (2002) states that, “too many students are too focused on their grades rather than on learning” (p. 2). Another problem for some high school seniors is that once they have a college acceptance they tend to carry over their poor habits of not studying much. It is expected that most college students should spend at least two hours of class preparation for every one hour spent in the classroom especially in mathematics. This means that a typical full time student should be spending 25 to 30 hours a week on homework, which does not seem to be happening in colleges across the country (Young, 2002).

According to Young, (2002) results from the latest National Survey of Student Engagement found that only 12 percent of last year’s freshmen at four-year residential colleges reported spending 26 or more hours per week preparing for classes, while the majority, 63 percent, said they spend 15 or fewer hours on class preparation which the survey defines as studying, reading, writing, rehearsing, and other activities related to your academic program. The most shocking statistic is
that nineteen percent of full time freshmen spend only 1 to 5 hours per week preparing for classes (p. 35).

It is obvious that there is a real problem with getting both high school and college students to complete their homework assignments. Researchers recommend the following steps for encouraging college students to study more (Young, 2002).

1. Require students to take study-skills courses or to attend orientation sessions that emphasize time management.

2. Involve faculty members in campus tours for prospective freshmen, to emphasize the importance of academics.

3. Better reward faculty members for teaching and spending time with undergraduates.

4. Provide more financial aid or otherwise encourage students to work fewer hours in jobs, to allow more time for studies.

5. Create “learning communities,” in which students are placed in groups of about 25 and share a set of classes to build a better sense of connection to the university and to the academic work.

6. Take steps to halt grade inflation.

In an attempt to motivate students to consistently complete homework assignments some schools have incorporated computer-based homework into the curriculum. Research has shown that computer-based homework systems are both praised and condemned (Brawner, 2000). Some critics say that the computer-based homework allows faculty members to be less involved with the student homework process. Advocates say that, “These systems are indispensable tools for grading homework for large number of students” (Brawner, 2000, p. 38). Computer-based homework is a time saver for faculty, which in turn allows them more time on task with their students. According to Bonham, Beichner, and Deardoff (2001), in a study of computer homework versus human graded homework in large introductory physics courses, the method of collecting and grading homework made little difference in student performance. In another study by Bonham, Beichner, and Deardoff (2003), they compared two groups of students of which one group completed homework online and the other used traditional methods. The performance of these groups in all aspects of the class shows that there was no significant difference.

**THE RESEARCH**

To obtain insight into students’ motivation and study habits concerning homework, especially online homework, I decided to interview four students taking a summer Algebra and Trigonometry course at a state college. It must be noted that all of these students have had mathematics problems of one form or another or they would not be taking a summer course.

I decided to use interviews (qualitative) rather than surveys (quantitative) to gain experience in the “Naturalistic Inquiry” form of research. This type of research is discussed in great detail in the work of Lincoln and Guba (1985) which I studied carefully and used the book as a guide during the study. The interview is especially useful when the researcher “does not know what he or she doesn’t know” (p. 209). With this approach the research design is allowed to emerge in that the results of previous interviews can affect the form succeeding of interviews. This adaptability is only one of the reasons why it is imperative to use a human as the instrument for gathering data in Naturalistic Inquiry “because only the human instrument has the characteristics necessary to cope with an indeterminate situation” (Lincoln, Guba, 1985, p. 193). The authors identify the following characteristics that uniquely qualify the human as the instrument of choice for naturalistic inquiry (p. 193):

1. Responsiveness,

2. Adaptability,

3. Holistic emphasis,

4. Knowledge base expansion,

5. Processual immediacy,

6. Opportunities for clarification and summation,

7. Opportunity to explore atypical or idiosyncratic responses.

Lincoln and Guba (1985) state that “for naturalistic inquiries the reporting mode of choice is the case study” (p. 357). The case study is the gathering and analysis of qualitative field research reports. The advantages of the case study format are (p. 361):

- Case studies may be written with different purposes in mind.
- Case studies may be written at different analytic levels.
Case studies will, depending on purpose and level, demand different action from the inquirer/writer.

Case studies will, depending on purpose and level, result in different products.

While case studies are usually used for complex, large-scale research, the approach is also appropriate for the present case.

**SETTING**

This research was carried out at paramilitary state college located in Southeastern Massachusetts where students must wear uniforms and are subjected to a regimented lifestyle that has a direct impact on academics. Another very unique aspect of this school is that every student must take mathematics through Calculus I regardless of his or her major. In all state colleges and universities entering freshman are required to take a mathematics placement exam called the Accuplacer. The results of this test are used to place students in their first mathematics course. On average approximately 50% of the freshmen class at this state college place into a remedial mathematics course, which is slightly higher than other state colleges but so are the mathematics requirements.

**THE MATHEMATICS CURRICULUM**

All mathematics classes are mostly lecture based explaining new concepts and the depth of demonstrating examples depends on the instructor. It is the goal of all instructors to prepare students for their next mathematics course whether it is Intermediate Algebra, Algebra and Trigonometry, or Calculus. For this reason all instructors who teach the same course use a common syllabus and give common tests. For the most part the syllabus is very fast paced and leaves no time for projects or group work during class.

In our college a major problem with teaching mathematics is that many students do not complete their assigned homework. For this reason the mathematics department decided to add a new online component to the remedial mathematics, Algebra and Trigonometry, Calculus I, and Calculus II curricula called MyMathLab, which is supported by the Addison-Wesley Publishing Company. The goal of adding this component was to give students another tool to help them with their homework, which would then hopefully result in success on tests and in the course. MyMathLab has online content that enhances and complements the pedagogy of the textbook. The components of MyMathLab are illustrated in Figure 1.

**HOW MYMATHLAB IS IMPLEMENTED**

MyMathLab homework consists of twelve weekly assignments worth ten points each. Every Thursday a new homework assignment consisting of approximately twenty to twenty five problems, including review problems are posted on the Internet. The students have eleven days to complete the homework assignment and submit them online. A useful feature of this approach is that students are allowed to try the problem over and over until they get the correct answer. If they cannot get the correct answer and don’t understand how to do the problem they can click on the “guided solution” which shows them step by step how to do that particular problem and then gives them a similar problem to do for credit. With this feature every conscientious student should receive a perfect score on his or her online homework. An interesting feature of MyMathLab is that teachers can determine how much time each student spends on homework.

MyMathLab along with the in-class lectures, the book, and written homework are all very important components in the mathematics curriculum and are in place to help mathematics student be successful in their mathematics course. The importance of homework is made clear to the students by counting it for 18% of their final grade. A summary of the grade point distribution is as follows: Four in-class tests (400 points), quizzes (150 points), online homework (120 points), written homework (100 points), attendance (50 points), and final exam (400 points). Common tests are given throughout the semester with faculty members taking turns writing the exams. The quizzes given in class are the responsibility of the individual instructors.

![Figure 1. The Features of MyMathLab](image-url)
It has been my experience that students who fail mathematics courses usually lose many homework points.

THE SUBJECTS

In a small summer school course (eight students) in Algebra and Trigonometry (the first credit-bearing mathematics course in the school), I persuaded four students to take the time for an interview on mathematics in general and homework in particular. As I have already mentioned these students are not typical because their previous mathematics course was either a remedial course (which they passed) or Algebra and Trigonometry, which they failed. The interviews were carried out either before or after a three-hour class and usually lasted about thirty minutes. Below I will describe the cases. The names of the students are not real and have been coded for consistency.

CASE STUDY #1

Kate is a 19-year-old sophomore from Colorado who attended high school at a boarding school in New England for three years. The last mathematics course Kate took in high school was Algebra and Trigonometry, which she took in her junior year. Before college Kate took a University Calculus course called Business Calculus which she passed but did not receive transfer credits for the course. Kate did poorly on the Accuplacer Exam, which placed her into the non-credit Intermediate Algebra remedial mathematics course.

Kate admitted that in high school she didn’t always attend her mathematics class because it was not her favorite subject. Homework, the even numbered problems without the answers in the back of the book, was assigned nightly and collected by the teachers. Even though Kate did not always attend mathematics classes she said she spent a lot of time doing her assigned homework.

Kate’s mathematics experience in college did not begin well. She failed her first semester of Intermediate Algebra, then passed Intermediate Algebra the second semester, and is currently taking Algebra and Trigonometry over the summer. She admitted that in the first semester, there were several factors that made mathematics difficult for her such as regimental duties and lack of sleep. When Kate was asked about her homework habits she stated that she spends about three or four hours a week on written mathematics homework. When she gets stuck on a homework problem she will go to a friend for help but will not skip over the problem. Kate also strongly stated that the written homework should not be collected because college students should be responsible to do their homework.

When Kate was asked about the online homework she replied “It either makes or breaks your grade, it’s an easy bunch of points to get because there is not a big time limit on it and I think it is a good opportunity to boost your grade.” She found that the guided solutions were very helpful when she didn’t understand a problem. Kate stated that the online homework component definitely enhanced the mathematics curriculum because it is “pretty much the same as the written homework and if you don’t understand a problem you can go to the guided solution.” When Kate was asked the question “If you had to choose between online homework and written homework what would it be?” Her response was “online homework because you have a week to do it and if you want to do it all on Monday night you can or you can spread it out throughout the week.” She also liked the instant gratification of knowing whether the answer typed in was right or wrong.

At the end of the interview I asked Kate if she had any suggestions on how to improve the mathematics program at the college. She had to think about this for a while but finally stated that the school accepts people with very low mathematics ability and that the school should raise the standards of students being admitted. This is an interesting comment from a student with mathematical difficulties.

CASE STUDY #2

David is a 21-year-old male who lived in Portugal until five years ago and is currently a junior. David attended high school in Portugal, and when asked about his mathematics experience there, he replied that it wasn’t very good. He stated that mathematics was taught differently than it is here, and that “the teachers lectured for an hour and if the class was talkative the teacher did not care, he/she just kept lecturing.” The homework load was very heavy and while David admitted spending a lot of time on the homework, he also admitted that he had great difficulty and often had to skip problems. When asked what he did when he didn’t understand something he replied, “in Portugal you don’t have the chance to ask teachers anything so you have to pay for a tutor but my family could only afford a tutor about twice a month.” David also stated that most children drop out of school in the eighth grade because everything is so expensive that many people can’t afford to send their children to high school.

When asked about his mathematics experience in college, he replied that it has been very good and that he does get people to help him. He said that mathematics
makes him nervous, and that he gives up easily and spends about two hours a week on written homework, but admits that he should spend more time. Usually when he doesn't understand a homework problem, he will skip it and not seek help; however, he does admit that this is due to lack of motivation. When David was asked if the written homework should be collected, he very quickly responded "yes." He said that if he knows the teacher is going to collect the homework, it motivates him to do it. If the teacher doesn't collect the homework, then he usually waits until the day before the test to attempt it.

While we were discussing the online homework component I sensed some frustration from David. He stated that he spends about three or four hours a week on online homework and was easily frustrated when the answer he entered into the computer was wrong. When I asked David what he does when he doesn't understand a problem on the online homework, he replied that he skips the problem. He does not use the guided solution or ask anyone for help. David stated that if he had a choice between written homework and online homework he would rather have written homework.

In wrapping up the interview with David I asked him if he had any suggestions on how to improve the mathematics program at the college, and his only reply was that the teachers should be more patient with the students and try to motivate them more. He has had some experiences where teachers have embarrassed him in front of the class due to a low test grade and told him that he didn't have a chance of passing the course.

**Case Study #3**

James is a 19-year-old sophomore who attended a public high school in Rockland, Massachusetts. James not only does well academically but is also a success on the soccer field where he was All Conference in his freshman year. When James was asked about his mathematics experience in high school, he explained that he was an A/B mathematics student and took mathematics all four years. His senior year he took a statistics course that he really liked. According to James homework problems were assigned daily and were either collected and graded or just checked for completion. He spent a lot of time doing homework throughout high school.

So far James' mathematics experiences in college have been good. He started in the Elementary Algebra class during the first semester and during the second semester he took Intermediate Algebra (both are non-credit remedial mathematics courses). He is currently enrolled in a summer Algebra and Trigonometry course. He says his mathematics experience has been good because he had the same teacher in both Elementary and Intermediate Algebra, and he really liked the way the teacher taught. James seems to like mathematics but states that at times it can be frustrating, but once you understand a concept he says it is a "great achievement."

When James was asked about the homework in college he said that homework is an important component of the mathematics curriculum. He spends about two to three hours a week on written homework and if he doesn't understand a problem, he will first look in the back of the book and see if he can figure it out from the answer. If this doesn't work he will look for a similar example in the book, and if both these methods fail he will either ask a friend or the teacher. He never leaves a problem that he doesn't understand. James likes it when homework is collected, but if it wasn't collected, he would still do it. He states that it is an easy way to earn points because he has to do it anyway. James likes the online homework but prefers the written homework. He finds the online homework frustrating at times when the answer is not input the way the computer wants it. For example, the computer may want the answer as a decimal, and if the answer is typed in as a fraction it will be marked wrong. James spends approximately three to four hours on the online homework, and when he gets stuck on a problem, he does find the guided solution helpful. When asked his preference between written homework and online homework, his reply was "written because the online is good homework, but a lot of times I will skip a lot of steps and do it in my head, but with the written homework, I write out all the steps, just so I know I don't make any mistakes." As James pointed out, these steps make a big difference on the tests. Although he prefers the written homework he states that the online homework definitely helps those students who complete the weekly assignments.

As the interview was winding down, I asked James if he had any suggestions on how to improve the mathematics program at the college. He replied that his experience has been good, but from listening to other students, he feels that some of the teachers need to slow down the pace and give more examples in the classroom.

**Case Study #4**

Chris is a 20-year-old from Brockton, Massachusetts who attended a Catholic High School for grades nine through twelve. Chris had a good experience with mathematics throughout his high school
career where he maintained a grade of B or C for most of his mathematics courses. He was given nightly homework, which he did most of the time. His teachers collected the homework, which he liked since it showed them what he was capable of doing.

Unfortunately, Chris’ mathematics experience in college was not the same as in high school. He started out in Algebra and Trigonometry in the fall of 2003, failed this course, and repeated the course in the spring of 2004 which he again failed. He is currently retaking Algebra and Trigonometry (for the third time) over the summer. His biggest complaint is that the teachers did not break steps down enough, and it was very easy to get lost. He repeated several times “it’s just that one step that you don’t get.” Not only did Chris have trouble with mathematics but he also admitted that he had a difficult time adjusting to the environment at school and meeting the demands placed on him. When Chris was asked about the written homework, he stated he had difficulty because he didn’t understand what was going on in the classroom. When asked what he would do about this he stated that, “I ask a friend or someone in my class for help.” He also stated that he did go to the instructor a few times. According to Chris, he spent about one or two hours a night on mathematics homework, and he did not like the fact that the teacher did not collect the homework. He stated strongly that when teachers collect the homework, it shows them what a student is capable of.

When the topic of online homework came up, Chris immediately stated that he “hates online homework.” When asked why, he said that the computer was too picky on how the answers need to be put into the computer, and he said that he spent more time trying to input the answer the correct way, and he found this to be very frustrating. According to Chris, he spends approximately six hours a week on the online homework. When Chris was asked if he thought the online homework concept in general was a good idea, he replied that it was a good idea if the computer wasn’t so picky. Chris did take advantage of the guided solution component of the software and found this to be very helpful. When asked which homework he preferred, either online or written, his response was “written because it prepares you for tests better since you have to show all the steps.”

As the interview was winding down Chris was asked if he had any suggestions to improve the mathematics program. His only response was “get better teachers.”

**CROSS CASE ANALYSIS**

Analyzing the interviews I found that all the students did homework every night and, three out of four students said that the nightly homework should be collected and graded. According to these students, when a teacher collects homework it motivates them to actually do the homework. Some students feel that collecting homework allows the teacher to track students’ progress and determine if he/she is falling behind. One student said that if the teacher does not collect the homework, then he (the student) waits until right before the test to complete the assignments, which is the path to failure in mathematics. The amount of time spent on written homework varied among the four students. Two students spend approximately two hours a week on written homework, one spends approximately three to four hours a week, and one student claimed he spends one or two hours a night doing homework which I find questionable since he failed Algebra and Trigonometry two times and is taking the course this summer for the third time. It was interesting to learn how the students dealt with the homework problems they did not understand. Two of the students would ask a friend or the instructor for help, and the other two students would make an attempt to find help but would not hesitate to skip that particular problem. Only one student said that he would use the examples in the book if he encountered difficulty. The other three students did not mentioned using the book for assistance.

Only one student liked the online homework and would rather do online homework then written homework. The three students who did not like the online homework thought that the general idea was good, but they didn’t like the program in use at the college. These three students felt frustrated at times because the MyMathLab software is very particular about how the answers must be typed into the computer. For example, if a student inputs the answer 2/6, the computer would mark it wrong because the answer should be in lowest terms. While the students find this frustrating, as a teacher, I find the online component to be an important teaching tool because it forces the students to follow instructions and to pay attention to details such as reducing fractions, making sure to round to the correct decimal place, and being able to incorporate parentheses into an answer when necessary. However, there are some legitimate complaints about the program. For example, if the answer to a problem is $x^2+3x+2$ and the student types the answer as $1x^2+3x+2$, the computer will mark this wrong, but not consistently. The students found that typing the answers the correct way was very
time consuming, and when a student gets an online homework problem incorrect, he/she is given a similar problem to do again. It is interesting to note that these three students spent more time on the online homework then they did on the written homework, but I wonder if most of the time was spent trying to type the answer in correctly. One of the features of MyMathLab that three out of four students found helpful was the guided solutions.

The last question that these students were asked was, “How can the mathematics program at this college be improved?” Two students said that the school should get better teachers (probably a common complaint among poor students). One student said that his experience has been positive because he had the same teacher for all his mathematics courses and he liked her. It was interesting to hear one of the students respond to this question, “the school should not accept students who have little mathematical ability.” That would mean that she should not have been accepted. What was encouraging is that the students did not suggest changing or eliminating homework as a component of the mathematics courses.

**SUMMARY**

In conducting this research three facts became apparent.

1. The students want the teachers to collect the written homework in mathematics because it motivates them to do homework. Unfortunately, from listening to the students it seems that many college teachers do not collect homework. One of the reasons for not collecting homework is a large number of students in classroom and the amount of time nightly homework grading takes.

2. It was also determined that the online homework component seems to have potential since all the students interviewed seemed to like the concept. One thing that this school might look into is different software that offers the same capabilities but is more user-friendly.

3. Most of the students do not spend enough time on their homework especially for students having difficulty with mathematics.

In hindsight, I have to mention certain limitations of the study. Originally, it was planned as a short-term pilot study to practice what I have learned from the book I was studying. To gain in-depth understanding of students’ perception about collecting and grading homework, more students with a wider range of ability must be interviewed. Interviewing more students would provide more data for more substantial conclusions. Sampling analysis and re-sampling is a fundamental part of naturalistic inquiry as diagramed by Lincoln and Guba (1985, p. 188). Therefore, re-interviewing the students after analyzing their original responses would offer more and/or other insights into the investigated issues.

**References**


ABSTRACT

A course graphic is a graphical depiction of the major concepts of a course and their interrelationships. It provides a visual framework for the students to understand what the major focus of the course is. If developed well, it can facilitate learning of material and the storage and retention of details. In this paper I summarize my experience as a finance professor in developing and using a course graphic in a graduate finance course.

“Words, words, words, I’m so sick of words!

I get words all day through, first from him now from you.

Is that all you blighters can do?”

From “Show Me” sung by Eliza Doolittle in My Fair Lady (Lerner and Loewe, 1956, side 2 band 4)

Traditional instruction in higher education has long been exclusively verbal: the professor’s lecture, the textbook descriptions, the writing assignments, and the examinations. The course graphic is an attempt to capture the structure of a course in an image. The image can then be used to help the student understand and remember more easily the relationships among the content topics. By being able to see the “big picture” of course content, students should be better able to learn and retain details of the course content. Research has shown that students have a variety of learning styles, so using graphics is important, particularly for visual learners.

WHAT IS A COURSE GRAPHIC?

A course graphic is a visual representation of course content that introduces the student to the structure by which the instructor has organized the course. It helps to separate the details from the concepts so that the students can see the framework on which the course is built. It takes advantage of the power of visual learning, which can be responsible for over 75% of what a student learns (Beaudry, 2000). It shows on one page how the key topics relate to each other. See Figure 1 for my course graphic.

WHY USE A COURSE GRAPHIC?

A course graphic takes a long list of topics about a course and develops an image showing how the topics relate to each other. Since many of my students do not have much context for the material that is to be covered in the class, explaining what the course is about can be tricky. The vocabulary of the subject means nothing to these students; it is a foreign language of sorts. Research shows that a key to successful acquisition of knowledge is organization (Beaudry, 2000), so a well-organized graphic can assist the student in mastering the course material.

Research in learning shows that students must be actively engaged in order to effectively learn (Woolfolk, 1993). A course graphic engages a student’s mind in a way that only an image can. A traditional syllabus usually employs only one mode of representation. When using a graphic and referring to it periodically, the instructor can assist the students in reviewing the key concepts of the course and the interrelationships.

With any new course that introduces the student to a subject in a completely new area for him or her, the material can seem quite overwhelming. New terms, new methods, new ways of analyzing things can swirl around the learner. In trying to master all of these factors, it becomes easy for the learner to lose the forest for the trees. In focusing on conquering the details, it is
difficult to see the framework of how the knowledge is integrated. By displaying the key concepts to be covered in the course, the instructor can help the student feel a greater likelihood of success that will increase his or her motivation (McMillan & Forsyth, 1991).

Writing and reading are linear processes; however, ideas, patterns and thoughts are non-linear (Hyerle, 2000). Using graphics enhances the learning of content by showing the content in a non-linear manner. Design guidelines for graphics suggest emphasizing the major topics (the big picture, so to speak) to help organize the material for students. With these images providing a framework, new material may be more easily connected to existing knowledge. (Woolfolk, 1993).

**LEARNING, MEMORY AND HEMISPHERIC SPECIALIZATION**

**LEARNING**

Learning is the process through which experience causes permanent change in behavior or knowledge (Woolfolk, 1993). There are two primary schools of thought regarding how people learn, the behavioral and cognitive schools, each of which encompass many individual theories and principles. Cognitive theory focuses on the internal mental activities that bring about a change in knowledge. They focus on mental activities such as thinking, remembering, creating and problem-solving. Behavioral theory focuses on the effects of external events on the person. Great scientists, like Pavlov and Skinner, looked at how external stimuli could produce observable responses.

The course graphic falls more in line with cognitive learning theories. The graphic attempts to stimulate cognitive processes, as outlined below, to help the student learn more effectively.

**MEMORY**

According to Woolfolk (1993), memory has three components: the sensory register, short-term memory, and long-term memory. The sensory register is the original source of input to the memory. It constantly receives input from all senses and retains all of this information briefly. It encodes what it perceives to be important and passes it along to short-term memory. Much of what we perceive is related to how we give meaning to sensory input. Many theories, such as Gestalt, bottom-up processing and top-down processing, indicate that people tend to organize sensory information into patterns and relationships for enhanced learning and storage.

The short-term memory can retain five to nine separate items at a time that will last approximately 20-30 seconds. Long-term memory holds information that has been learned well. It has unlimited capacity and duration, although information can take some time to be learned well enough to be stored here. The brain is capable of absorbing more than 36,000 images per hour (Hyerle, 2000).

Woolfolk (1993) cites Paivio who suggests that information is stored in long-term memory as a visual image, verbal unit, or both; information that is coded both visually and verbally, as the course graphic attempts to do, is easiest to remember. Woolfolk also cites Craik and Lockart, who have an alternative view of memory from the three-component model above. Craik and Lockart suggest that what is remembered is related to how the information is analyzed and connected with other information; the more the person processes the information, the better the recall of it.

The course graphic can provide the framework to help students analyze the key topics in the course and interconnect them. Careful design of the graphic and periodic review of it by the students creates the familiar image that organizes information and becomes memorable.

**HEMISPHERIC SPECIALIZATION**

For decades, scientists have studied the brain to understand how it functions with respect to processing information. As technology has advanced, scientists have been able to do more sophisticated research such as knowing that certain areas of the brain control various processes in the body.

Buzan cites many research studies conducted in the 1960’s and 1970’s, especially work done by Nobel prize winner Roger Sperry, Robert Ornstein, and Eran Zaidel. In summary, the brain has two halves that are connected by a complex network of nerve fibers. Initial research concluded that each hemisphere specialized in different types of mental activity. In most people, the left cortex deals with logic, words, numbers and reasoning, “the so-called academic activities” (Buzan, p.17). The right cortex deals with images, imagination and patterns. While one side is actively processing information, the other side tends to rest. Research showed that when people worked to develop weak mental areas, that all mental performance seemed to improve.

Further research has discovered that each side of the brain actually replicates to a large degree the other side’s abilities. Each hemisphere is capable of wider and subtler mental activities than previously thought. Both
Perecman (1983) and Springer and Deutsch (1998) find no evidence that only one side of the brain is involved in a given cognitive task. Instead, both sides are engaged during mental processes, even though one hemisphere might be more dominant in a particular process.

The implications for the course graphic are that it appeals to an area of brain function that is not as active during typical academic work. Like any muscle, the more the brain is exercised, the more it develops, leading to an increase in the capability to learn and remember.

MY COURSE

The course that I teach is “Financial Management for the Non-Financial Professional.” It is a graduate level course intended to give professionals an introduction to financial management. This course was originally designed to be part of a master's degree program in the College of Management for manufacturing professionals. At the University of Massachusetts Lowell, engineering doctoral candidates must take three electives in management science, and this course is a popular choice. The students benefit from this course because it teaches these professionals how to understand financial statements and use the information to run their departments better. It also shows how and why financial managers operate, thus improving the relationship between the professions. Last, in the post-Enron era, this course emphasizes the increased importance of accurate record-keeping, full disclosure, and ethics.

Teaching this course is a challenging endeavor. The students that tend to take this course have diverse backgrounds. Many students are from technical fields. The ethnic backgrounds vary as well; typically, each class has students representing at least eight different countries of origin.

Students have varying degrees of comfort with the English language, so their understanding of my lectures may vary. Although I have 20 years of experience in working with people across the world, there are times when it is difficult for me to understand a question that a student is posing because of their English pronunciation skills. It may take a few exchanges before I can figure out what their problem really is and how to answer it in a way they can understand better. Different cultural backgrounds make new subjects even more confusing. For example, when talking about 401K plans to illustrate payroll deductions, some students have no concept of what a 401K plan is, thus I need to choose my examples carefully. In some cultures, students are less prone to ask questions of the instructor, especially females. Also, the thinking framework is different. While in some technical situations, things either work or they don’t. In business whether things work or not may depend on the situation itself. This becomes frustrating for the students who want to know rules that work every time.

DEVELOPING MY COURSE GRAPHIC

Keeping it simple, making it relevant, making it accurately reflect the relationships, having it effectively use color and fonts and shapes—these are some of the details that continually run through my mind when developing a course graphic. The task of developing the graphic itself can be daunting.

Developing a course graphic is very similar to developing a brand new course. I think about the desired learning outcomes, the topics to be studied, and the sequence in which they will be presented. I have to evaluate how the course fits into the overall curriculum to avoid redundancy and present appropriate material in sufficient depth. I consider the typical student profile and why they are taking the course. I also take into consideration the kinds of jobs the students will or do have so that they learn content that will be useful and applicable in their job.

According to Miller (2003), the primary design elements to think about in creating an effective course graphic are the following:

1. Simplicity: Identify three to five key concepts of the subject and the most important subordinate concepts to keep the graphic simple.

2. Organization: Represent the relationships of the key concepts and their subordinates in an organized manner; seek to use patterns to reinforce the perception of relationship.

3. Readability and precision: Use consistent font sizes and typefaces, color and shapes to enhance readability and precision; avoid clutter; use uniform terms.

My first task is to examine the course topics and student learning objectives in detail. Because it is an introductory graduate level course, the content of the course is quite comprehensive. My focus is mostly on declarative and procedural knowledge. The textbook that I use for this course, Accounting: What the Numbers Mean by Marshall, McManus and Viele, separates the material into two sections: Financial Accounting and Managerial Accounting. Because the course is intended
tool because it has a number of shapes available in its

Once I have settled on the topics and their sequence, I group the topics so that the number of groups displayed on the graphic is reasonable. This is challenging for me because I want to avoid making things very simple and generic, but rather show the graphic and relationships that are not necessarily memorable. Yet if there are too many elements on the page, the graphic is too busy and difficult to remember.

After drafting the design, I try to view it from the students’ perspective. Lowe (1993) outlines several questions that a student may ask including “what is this diagram trying to explain to me and how am I supposed to make use of it to help me learn?” (p. 44). I practice presenting this diagram to make sure I can explain it clearly. Since most of my students have never been exposed to a course graphic, I need to describe how it can be used and its benefits. I emphasize that visual images of the content (concept) provide another representation of the course ideas and their relationships and helps to see them from another angle.

I settled on having three main concepts for my course: Transactions (the recording of data), Financial Statements, and Financial Analysis. Typically transactions are included in Financial Accounting because they are necessary to produce the financial reports. However, in my view, transactions are also critical to performing accurate financial analysis, so I felt it was more appropriate showing this as a separate concept with a relationship to both financial statements and financial analysis.

I identified main topics under each concept. Along with the concepts and topics, I included some key words that would differentiate each concept from the other and make them more understandable for the students who are new to the subject. One key differentiator is the timeframe; transactions are done in the present, financial statements reflect historic results and analysis focuses mostly on the future. The other key differentiator is the primary audience; financial statements are primarily oriented to someone external to the company, and financial analysis is primarily oriented to someone internal to the company.¹

For this graphic, I used Microsoft’s PowerPoint tool because it has a number of shapes available in its drawing set. The shapes that I decided to use for the concepts were ones that could physically fit together. I did not want to use mundane shapes (such as squares or circles) if I could avoid it, mostly because I like to use something that is fresh to the eye. The transactions are in the bottom shape since they support the other two main elements of the course. In addition to fitting with the triangle of the transactions, I chose the trapezoids because they are congruent and create the image of a book when lined up together. I centered the text about the concepts (in bold text) and topics. I decided to put the differentiators along the sides of the trapezoid (in rectangles) to make them look different than the concepts. I am using only black and white for practical reasons; we have no color copiers for our syllabi, nor do I want to justify the expense of reproducing them in color. Eventually I may make my electronic copy in color, but for now, I prefer to be consistent with the copies the students have. See Figure 1 (p. 44) for my course graphic.

**USING MY COURSE GRAPHIC**

I incorporate the course graphic into my syllabus and into several lectures throughout the course. It is useful for non-finance majors because it helps them to monitor the content and our progress through the course. As new terms are introduced it can help the students to put the terms into a framework and notice the relationships.

I include the course graphic in my syllabus so that the students have a copy of it. It serves as a roadmap for the content of the course. To me, it makes for a more interesting review of how the content will be presented rather than reviewing a calendar which says, “week one we do chapter one; in weeks two and three, we’ll cover chapter two; etc.” Since one strategy for memorizing information is repetition (Munter & Russell, 2002; Olgren, 1998), presenting it to students on the first day becomes their first exposure to it.

Throughout the course, I display the graphic at the beginning of the class to remind students about where we are and how it fits into our overall plan. It gives me the opportunity to remind the students about the topics we have already studied and how they have related to each other. At times, I ask the students to explain what they think about grouping certain topics together. As they learn the material, the relationships begin to make sense and the picture becomes an image to which

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¹ Yes, it is true that outside analysts do analysis tasks and internal managers look at financial statements, but I am taking the perspective with students about primary audience, not every audience.
they can relate. Since they have a copy in their syllabus, it is easy for them to locate the topic (or concept) and annotate it each time we review it. The image reinforces learning and serves as a memory aid (Beaudry, 2000).

To involve students and assess their understanding, I display the graphic in different lectures and ask questions about the relationships. Their responses tell me how effective the graphic is at making its point. If students have a difficult time recalling relationships and cannot construct the relationship on their own, they may be having trouble with the material. If this is the case, I try to trace where the difficulty lies and plan to resolve it. If the failure for recall is due to the lack of effectiveness of the image, then I try to rethink the course graphic to make it work for these students. Student verbal and non-verbal reactions to my descriptions of the graphic help me in tailoring the graphic more effectively.

There are other methods for engaging students with the graphic, such as giving them one with blanks to fill in. I decided that using the graphic as described above was the best method to use with my students. It makes use of repetition and ties things back to the key concepts. In my experience, my graduate students tend to be very motivated to learn, so I try to respect their natural motivation and not overemphasize things that people understand quickly.

CONCLUSION

I have experienced success in using my course graphic in several courses. Students have made worthwhile remarks indicating that the graphic has aided their learning of the material. The exercise of putting the graphic together has made me think of my course differently, analyzing my assumptions and questioning what the real value of the course is to the student. The course graphic has become my framework to ensure that I emphasize the key concepts and make clear to the students how the concepts relate to each other.

References


Developing New Mathematics Curriculum
Andrew Golay, Lowell High School

Last summer I had the opportunity to work for the Education Development Center (EDC) in Newton, Massachusetts, developing a high-school curriculum. In this article, I would like to share some of the main features of the curriculum. The curriculum is presently called the Center for Mathematics Education Project (CMEP), and it contains four courses: Algebra 1, Geometry, Algebra 2, and an equivalent to pre-calculus. The Algebra 1 course is being field-tested this year nationwide by teachers in several states, including Massachusetts, New Hampshire, Colorado, California, and Washington State. Following are some ingredients to their curriculum design that I found particularly interesting.

GUESS, CHECK, REFINE

Consider the following two problems:

1. Juan rides his bike up a hill at 2 miles per hour, for 20 minutes. He gets to the top then takes a shortcut down the hill, going 10 miles per hour for 2 minutes. What was the total distance of his trip?

2. Juan rides his bike up a hill for 20 minutes. He gets to the top then takes a shortcut down the hill, going 5 times as fast, for 2 minutes. In total (up and down), the distance he rode was 1 mile. How fast was he riding up the hill, in miles per hour?

Taking a minute to think about which problem is more difficult reveals that although both problems in a sense represent the same equation, the first problem is easier. Students can just crunch the numbers:

Distance up the hill= rate $\times$ time = $2 \frac{mi}{hr} \times \frac{1}{3} hr = \frac{2}{3} mi$.

Distance down the hill= rate $\times$ time = $10 \frac{mi}{hr} \times \frac{1}{30} hr = \frac{1}{3} mi$.

Total distance = distance up + distance down = $\frac{2}{3} mi + \frac{1}{3} mi = 1 mi$.

If students try doing that with the second problem, they’ll have trouble because they don’t have an explicit formula for the uphill speed. Rate = distance/time won’t work, because the uphill distance is unknown. Advanced problem solvers can formulate the equation:

$rate \times \frac{1}{3} hr + (5\text{-rate}) \cdot \frac{1}{30} hr = 1 mi$,

where r is the time riding up the hill. On the other hand, novices who aren’t familiar with variables may have considerable trouble. So, they are encouraged to take a guess about how fast Juan is riding up the hill. It doesn’t matter what the guess is, it’s what is done with the guess that counts. If students guess 6 miles per hour, then the distances up and down the hill are

Distance up= rate $\times$ time = $6 \frac{mi}{hr} \times \frac{1}{3} hr = \frac{6}{3} mi$.

Distance down= rate $\times$ time = $5 \cdot 6 \frac{mi}{hr} \times \frac{1}{30} hr = \frac{5\cdot 6}{30} mi$.

The distances can then be tested (they should sum to 1 mile):

$\frac{6}{3} mi + \frac{5\cdot 6}{30} mi = 1 mi$?

This equation is false, because the left side equals 3mi. The guess was a little too high. That’s okay, though. The important part is that students keep track of their steps. If the guess is $4 \frac{mi}{hr}$ the equation is:

$\frac{4}{3} mi + \frac{5\cdot 4}{30} mi = 1 mi$.

and so on. Students will eventually see a pattern that generalizes to the following statement: If he went r miles per hour up the hill, then the equation (with the “mi” clutter gone) is

$\frac{r}{3} + \frac{5r}{30} = 1$,

and then student just need to solve for the value of r that makes the equation true. In this case, the process of formulating an equation can be just as valuable as finding the value of r.

According to Al Cuoco (personal conversation), who has used this method during his 30 years of teaching, the most difficult thing for his students was taking a guess, because they were afraid to guess the wrong answer. Once they overcame the initial fear, his students were able to use the guess-check-refine (or, aptly called, “guess-check-keep track of your steps”) cycle, with relative ease. See Cuoco (1993) for a theoretical framework for this concept.

EQUATIONS AS POINT-TESTERS

During my student teaching last year, I noticed that my students tended to have trouble with concepts like
slopes and equations of lines. One reason for this could be that term “slope of a line” is introduced at an unwarrantedly early time. Many teachers assume that a “Cartesian Connection” (Knuth, 2000) between equations, slope, and lines is obvious, but I think from a student’s eyes, an act of force-feeding might be occurring, rather than conceptual understanding being developed. If a teacher simply asks a student to “find the slope of this line,” there is no forum for questions like the following to be answered: “Does a line have a slope?” “What does slope mean geometrically?” “Why does a line have ‘a’ slope and not more than one?” Since a defining characteristic of a line through a given point is that you can pick any two points on the line and the slope between them will be equal to the slope between any other two points on the line, why not use that as a foundation? That’s exactly the approach of the CMPEP authors: Instead of starting with the slope of a line, students are taught to find the slope between two points, which can be used to build equations of lines as point-testers. We can see if three points, A, B, and C are collinear (that is, if they lie on the same line) by the following “test”:

1. Take the slope between two of the points, say A and B.

2. Take the slope between A and C. If it’s equal to the slope between A and B, then C is on the same line as A and B, which can be visualized if teachers choose the slope and points wisely.

Suppose A= (1,2) and C= (3,4). Then the slope between A and C is

$$\frac{4-2}{3-1} = \frac{2}{2}$$

If students want to check that some point, say (x, y), has that same slope between it and A, they can make a “point-tester” equation:

$$\frac{y-2}{x-1} = \frac{2}{2}$$

The point-tester idea generalizes to many situations, even beyond equations. For example, the closed region bounded by the lines x= 1, x= 2, y= 5, and y= -7, has this point tester:

1 \leq x \leq 2,
-7 \leq y \leq 5

So, students can quickly find that (0,1) is not in the region because the x-coordinate falls outside of the acceptance region for x. In summary of the point-tester idea, a set of points can be characterized using a “point-tester” equation or inequality as follows: A point is part of the set of points being characterized if and only if substituting the values of x, y, and/or z for points into the equation or inequality will make the equation or inequality true.

EXTENSION

As students explore mathematics during and beyond Algebra 1, they will notice that different types of mathematical objects exist, each of which can form reasonable algebraic structures.

Arguably, the most well known mathematical objects are numbers, and the most accessible types of numbers are the counting numbers: 1, 2, 3, 4, and so on. Together with their negatives and zero, we have the set of integers: {..., -3, -2, -1, 0, 1, 2, 3, ...}. The set of integers is classically denoted Z from the German word zählen, for “numbers”. We can add two integers together, and the result, called their sum, is another integer. For example, 1 + (-5) = -4 is an integer. We can also multiply integers by one another, and the result, called their product, is also an integer. This can be explained by the fact that multiplication by an integer is actually repeated addition. For example, 4*5 is actually 5+5+5+5. Addition and multiplication are called operations.

Two other operations are subtraction and division. They have very special roles, because they can be used to undo addition and multiplication, respectively. That is, x minus y is the number you add to y in order to get x. Likewise, x divided by y is the number you have to multiply y by to get x. The results of subtraction and division are called differences and quotients, respectively. Notice this: The difference between two integers is always an integer (think of it as measuring the directed distance between two rungs on a ladder), but the quotient when dividing an integer by another is not necessarily an integer. For example, 7 divided by 2 is not an integer. If it were, then it would be between 3 and 4, since 3*2=6 and 4*2 = 8. But there aren’t any integers between 3 and 4, so either we can’t divide 7 by 2, or we need to extend our concept of number. Let’s extend.

But first one caveat: Notice that zero is the product of any integer and zero. If we let a be a nonzero integer and say that a divided by b is the number we multiply by b to get a, and then let b equal zero, it implies that the product of a divided by b and zero is a. But the product of anything and zero is zero, so a must equal zero, a contradiction. So, doing something like 25 divided by zero makes no sense, because it produces said contradiction. Even zero divided by zero is problematic, because if zero
divided by zero is $x$, then the product of zero and $x$ is zero, which gives us no information about $x$. Hence, zero divided by zero is undefined. Therefore, we avoid division by zero altogether, because it allows us to make egregiously false statements.

The set of values of $m$ divided by $n$, where $m$ and $n$ each can be any integer, except that $n$ doesn’t equal zero, is denoted $\mathbb{Q}$ (the Q stands for quotient) and called the set of rational numbers. They’re nice, but then what if we want to find the length of the diagonal of a square whose sides have length 1? According to Pythagoras, we get the length of the diagonal to be the number $x$ such that the product of $x$ and $x$ is 2 (try it yourself). That number, nowadays represented by the symbol $\sqrt{2}$, can measure distance on a number line, but it isn’t rational.

Here’s a quick proof:

Suppose $\sqrt{2}$ is rational. Then it equals some integer $m$ divided by another integer $n$. Another way to say this is that we can multiply $n$ by $\sqrt{2}$ and get $m$. So $\sqrt{2}n = m$. We can assume $m$ and $n$ have no factors in common except 1, because if they did, we could just divide both sides of the equation by them and get a similar equation. Squaring both sides gives $2n^2 = m^2$, which means $m^2$ is even. So $m$ can’t be odd, since if $m$ is odd, $m^2$ is odd. Hence, $m$ is even (2 is a factor of $m$). So, $m = 2k$ for some integer $k$. Substitution gives $2n^2 = (2k)^2$ and therefore $2n^2 = 4k^2$, and finally $n^2 = 2k^2$, which implies $n$ is even, and so is $n$. That is, 2 is a factor of $n$. Hmm… 2 is a factor of both $m$ and $n$. But we said $m$ and $n$ have no factors in common except 1. We followed all the rules but got a contradiction. The only thing that was questionable in our reasoning was the original supposition that $\sqrt{2}$ is rational. We have no choice but to conclude the opposite: $\sqrt{2}$ is not rational.

As it turns out, the square root of any prime number is not rational, and even more powerfully, the square root of any number that is not a perfect square (like 9, 16, 25, etc.) is not rational. However, these numbers can be used to measure distance, just as $\sqrt{2}$ measures the length of the diagonal of a square with side length 1. So there are even more numbers out there that may measure length on a number line but that may or may not be expressible as the ratio of two integers. The set of such numbers, called $\mathbb{R}$ (the real numbers), can essentially be defined well enough for a high school curriculum as the set of numbers that can be represented by points on a number line.

So, in the CMEP curriculum, students are introduced to numbers in these stages, not necessarily using the examples I used above, but in the order: integers, then rationals, then real numbers. A clear attempt is made to introduce each new set of numbers as an extension of the smaller one preceding it. Eventually, students learn about complex numbers as an algebraic structure in its own right, as well as several other types of algebraic structures using objects besides the three sets mentioned above. For example, students learn how to multiply and add polynomials, matrices, geometric figures, complex numbers, angles, permutations, and more, learning that such objects can act as algebraic structures in their own right.

**CONCLUSION**

I’ve only mentioned a few of the many interesting design features of the CME Project. Other features are student dialogues, different representations for different purposes, “try it with numbers”, form implies function (also known as the if it walks like a duck and quacks like a duck, it might well be a duck principle), and many more. The curriculum is designed with traditional course and concept names, for teacher and parent accessibility, while applying research-informed pedagogy, such as that used in countries with highly successful academic records. The highly successful middle school Connected Math Program (CMP) is a different project with different authors, but CMEP can definitely be considered to complement the former at a high-school level. Its expected year of completion is 2008, and hopefully the nation’s high schools will thus be affected in a positive way.

**References:**


Call for Papers

Educators, researchers and graduate students are invited to submit papers that will be presented at the Tenth Annual Colloquium on Research in Mathematics and Science Education and published in the Colloquium Journal, vol. X. The papers must discuss issues and trends in Mathematics and Science Education.

WHEN SUBMITTING A PAPER, PLEASE USE THE FOLLOWING GUIDELINES.

1. Submit an electronic version of the paper and one hard copy, an abstract, approximately 150 words, and a biographical sketch, about 30 words. All pictures and diagrams must be submitted in a separate document.

2. Use double spacing with one-inch margins.

3. For references, diagrams, etc. follow the style described in the Publication Manual of the American Psychological Association (APA), Fourth Edition.

4. Paper length must not exceed 30 pages, including pictures, tables, figures, and list of references.

5. Paper must be received by November 15, 2004.

6. Authors will be notified about the status of their paper by January 15, 2005.

7. A Colloquium will be scheduled for April 2005.

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